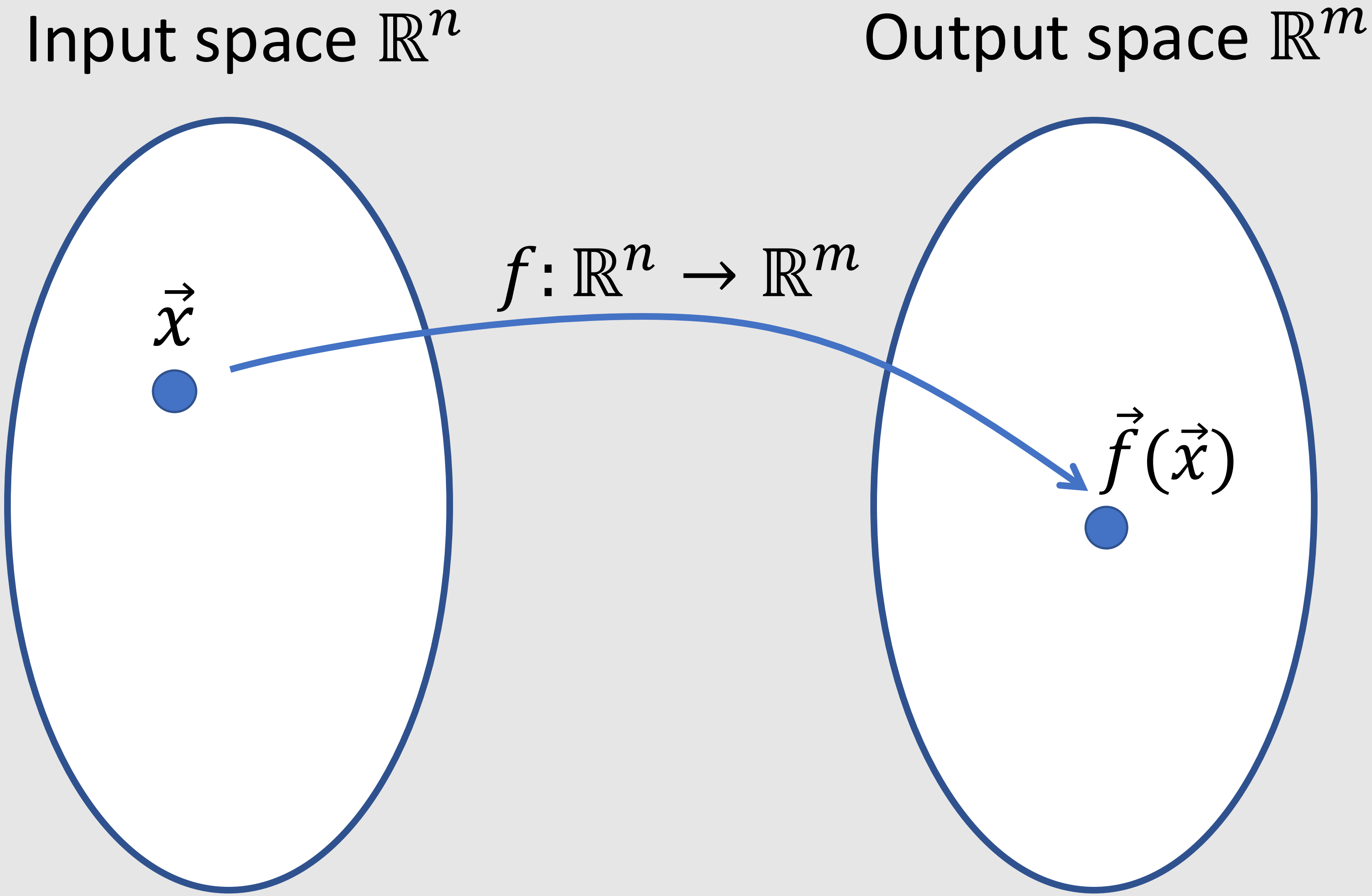
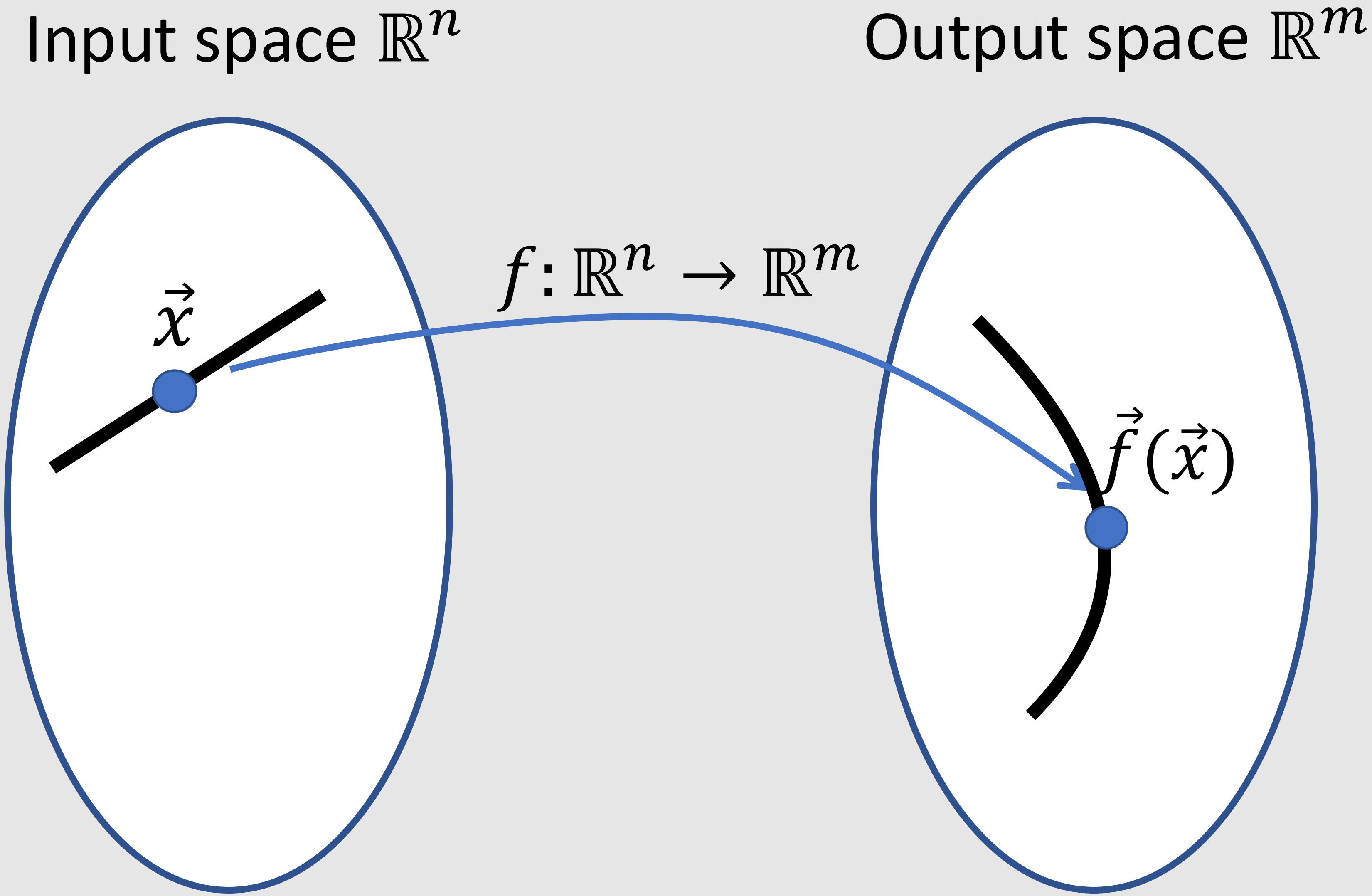
Jacobian & Hessian

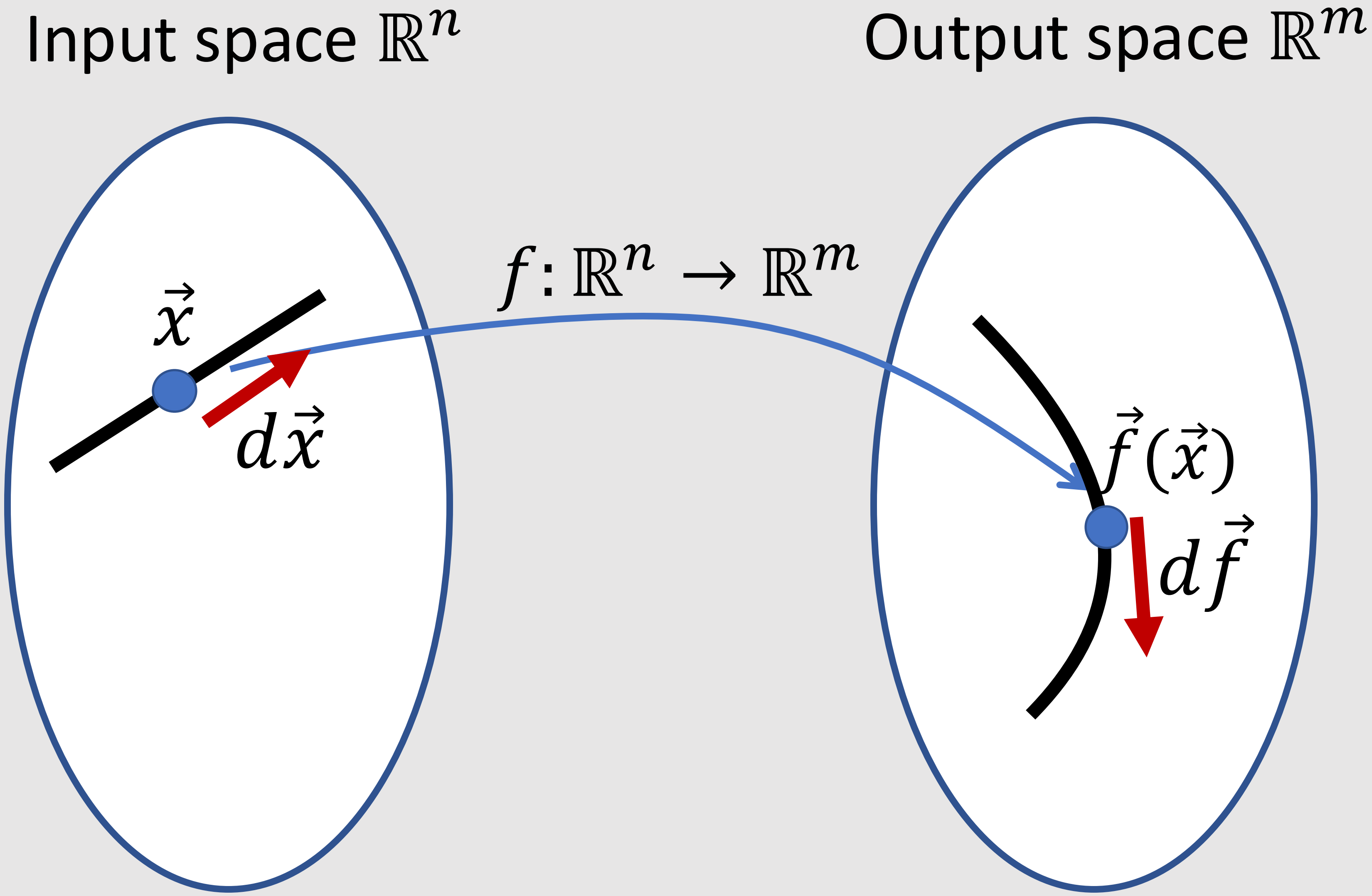
Multivariate Function: High Dimensional Map



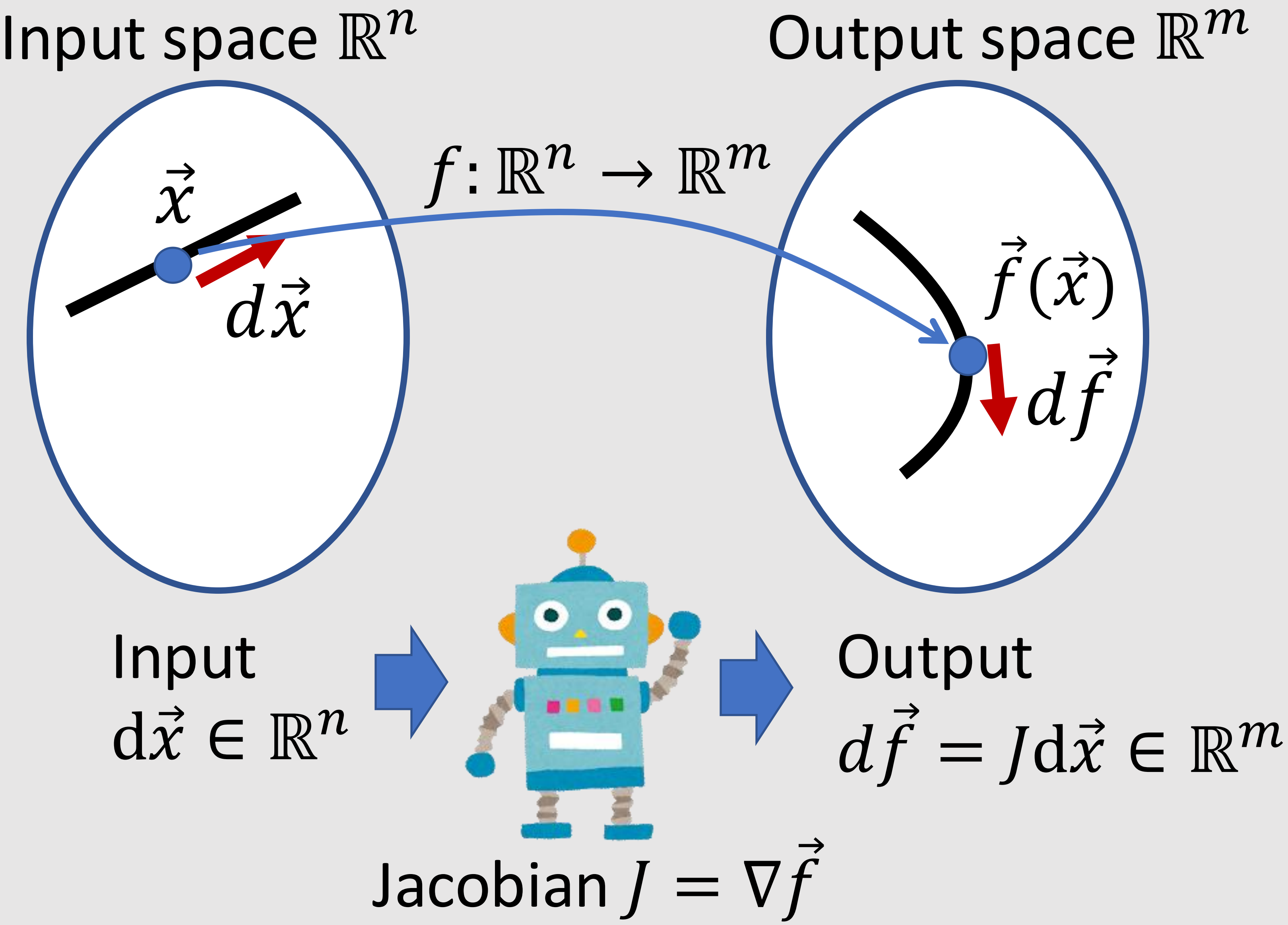
Trajectory of the Function



Differentiation of the Map

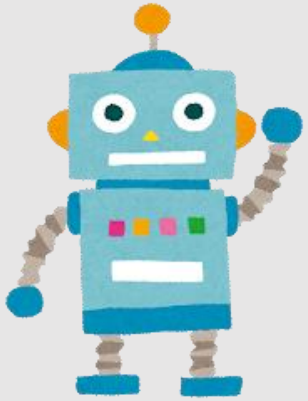


Jacobian Matrix: Gradient of Map



Jacobian Matrix: Gradient of Map

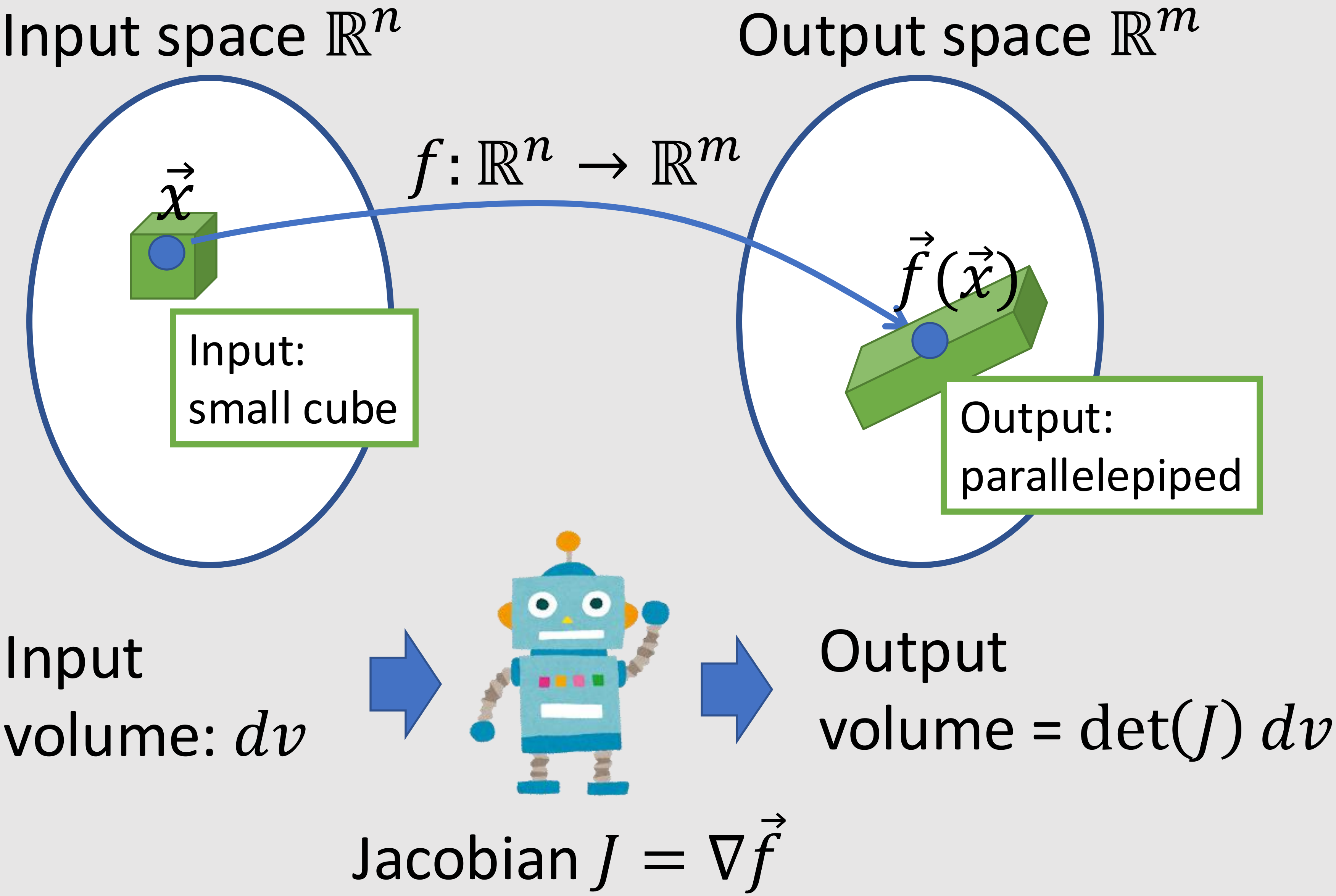
Input
$\mathrm{d}\vec{x} \in \mathbb{R}^n$



Output
$d\vec{f} = J\mathrm{d}\vec{x} \in \mathbb{R}^m$

Jacobian $J = \nabla\vec{f}$

Jacobian Determinant: Volume Change Ratio

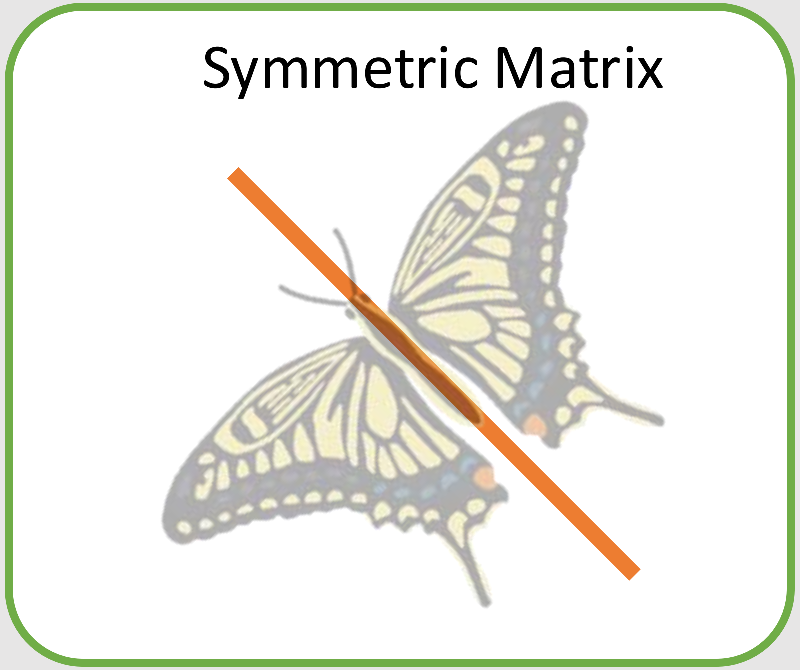


Hessian Matrix: Jacobian Matrix for Gradient

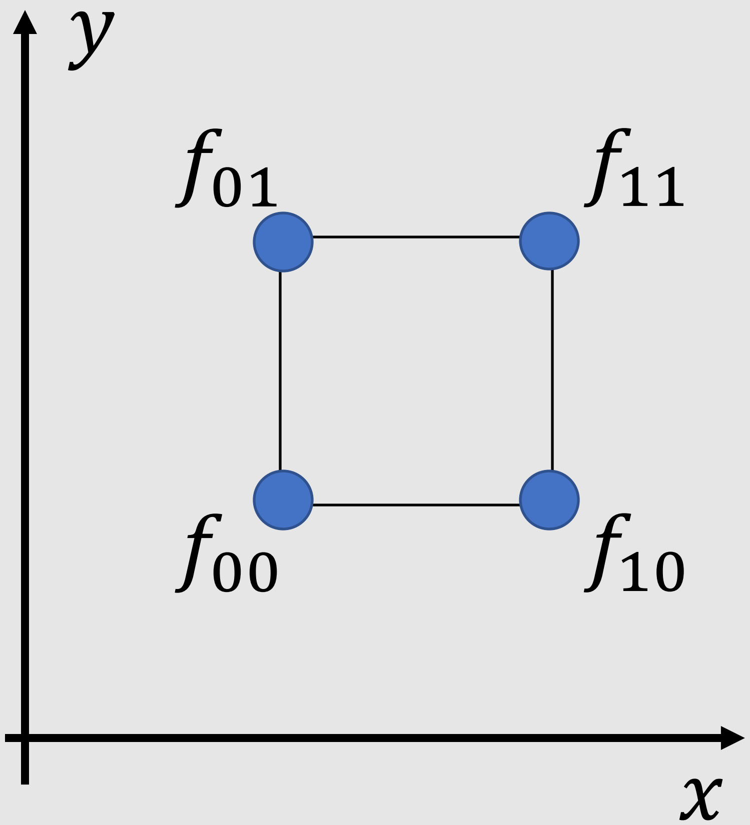
- Second derivative of a scalar function $f(\vec{x})$

$$\boldsymbol{H}_f = \boldsymbol{J}(\nabla f(\vec{x}))$$

Symmetricity of Hessian



- Hessian is symmetric if $f(\vec{x})$ is continuous



$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) \approx (f_{11} - f_{10}) - (f_{01} - f_{00})$$

equal

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) \approx (f_{11} - f_{01}) - (f_{10} - f_{00})$$