# Collision Detection

衝突検出

# Applications

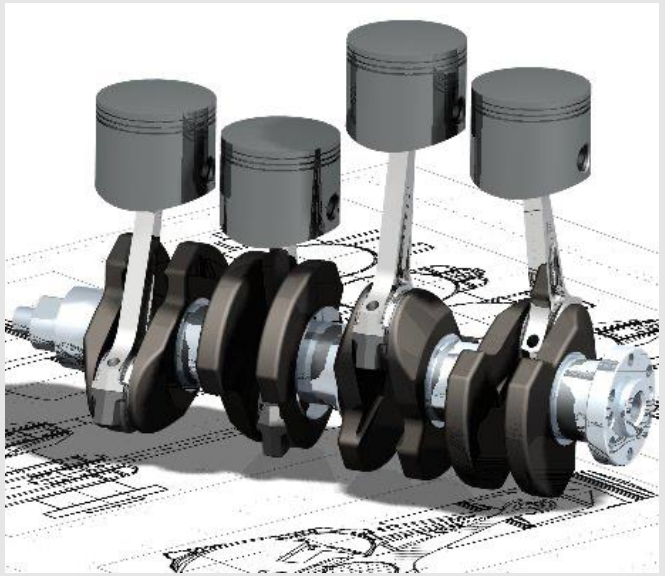
## Computer Graphics

(Wikipedia)

## Robotics

(Wikipedia)

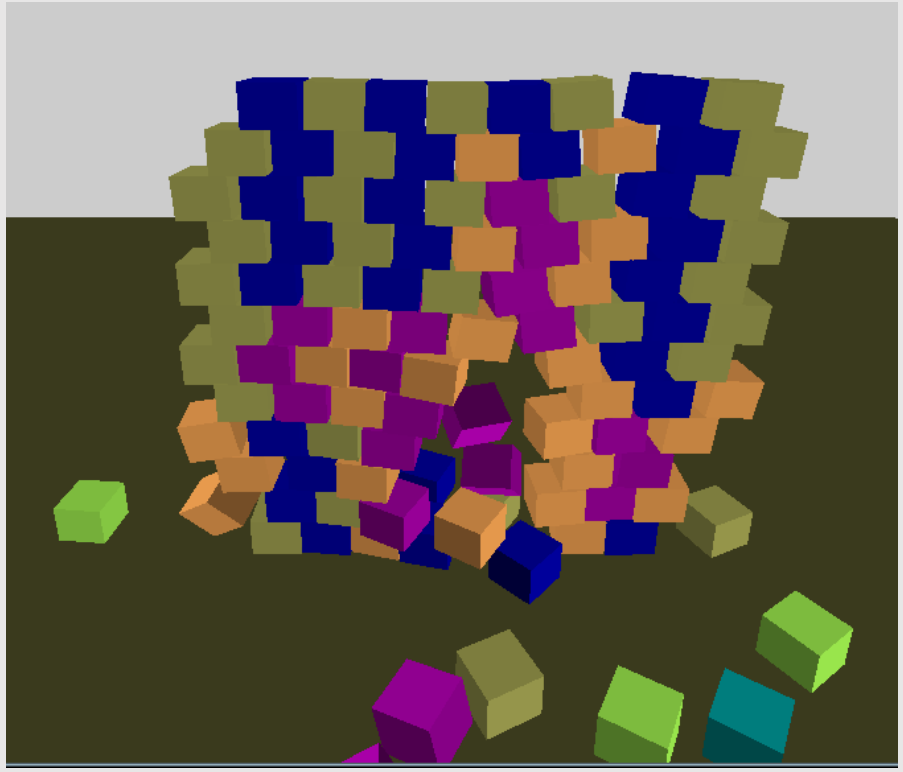
## CAD



(Credit: freeformer @ Wikipedia)

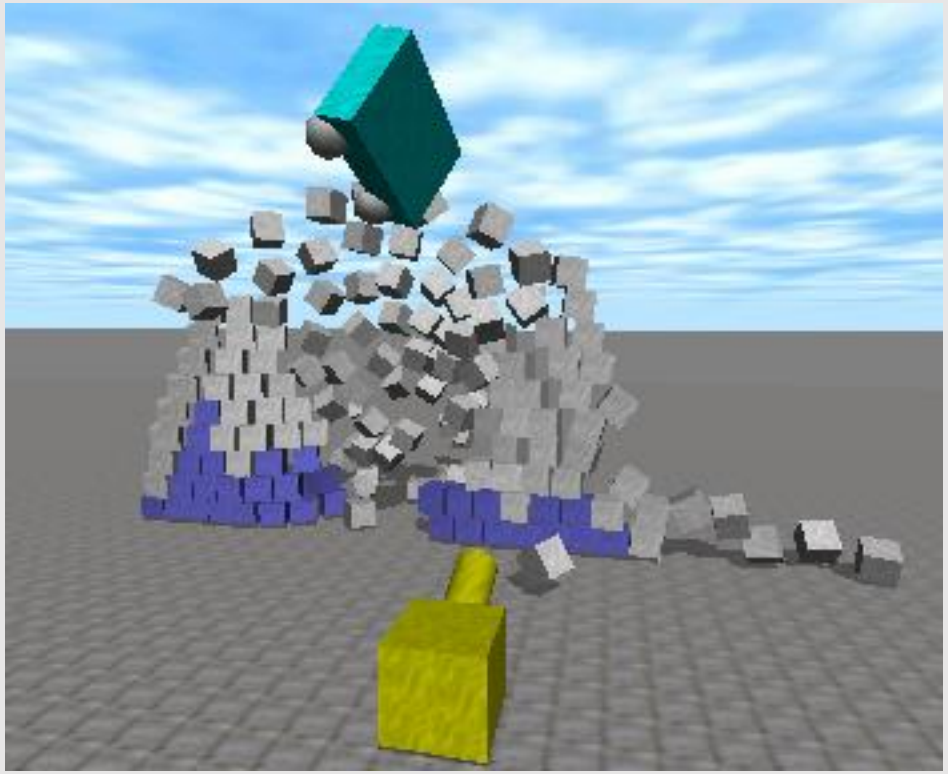
# Popular Rigid Body Simulation Engine

Bullet



(Credit: SteveBaker at Wikipedia)

Open Dynamic Engine



(Credit: Kborer at Wikipedia)

# Real-time Collision Detection using GPU


**Vivace: a Practical Gauss-Seidel Method for Stable Soft Body Dynamics**
Marco Fratarcangeli, Valentina Tibaldo, Fabio Pellacini
ACM Transactions on Graphics (SIGGRAPH Asia), 2016
http://www.cse.chalmers.se/~marcof

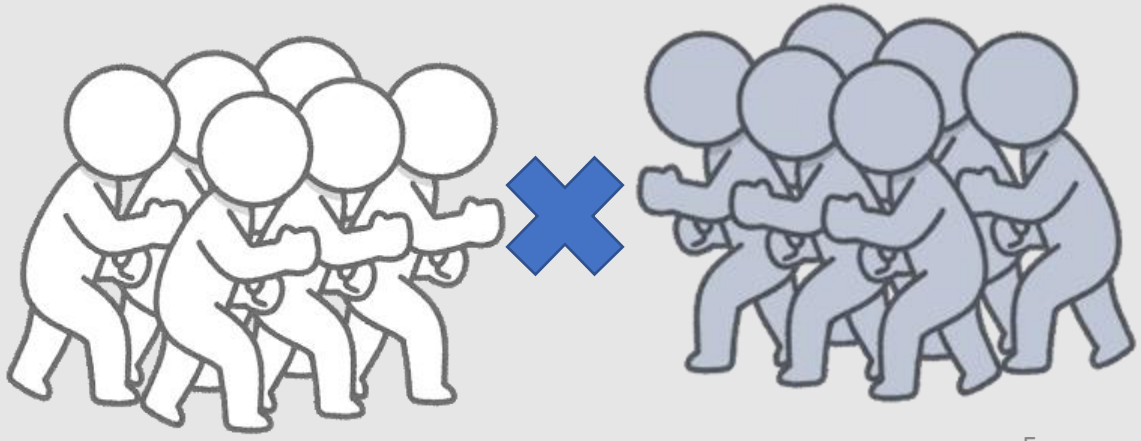
# Brute-force Collision Detection Never Works

- If there are N objects, there are N(N-1)/2 number of pair

→ $\mathcal{O}(N^2)$ complexity is too slow!

$\mathcal{O}(N)$

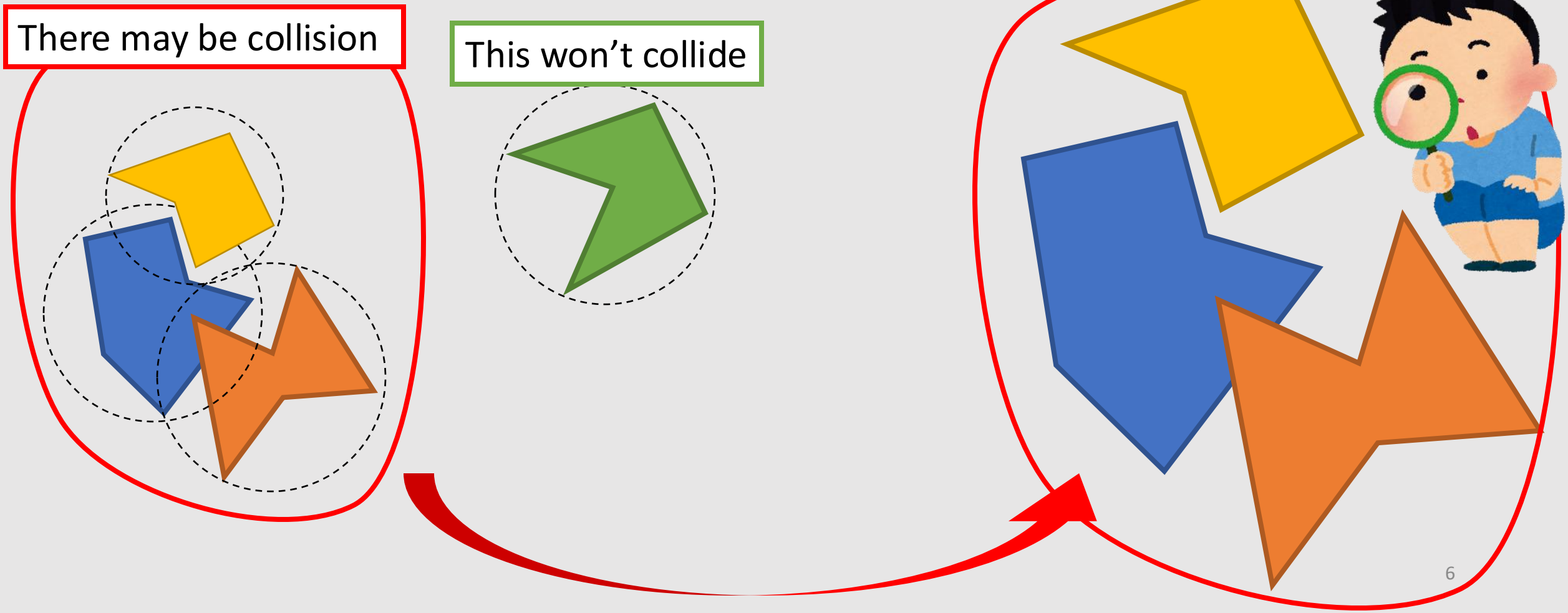
$\mathcal{O}(N^2)$



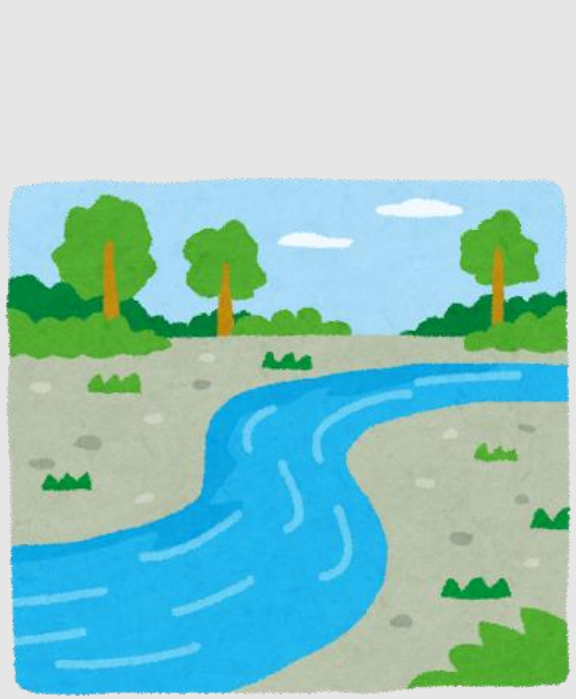
# Collision Detection in Two Stages

**Broad Phase**: extract candidate

**Narrow Phase**: actual check



# Idea of Finding Collision (like a Garimpeiro)

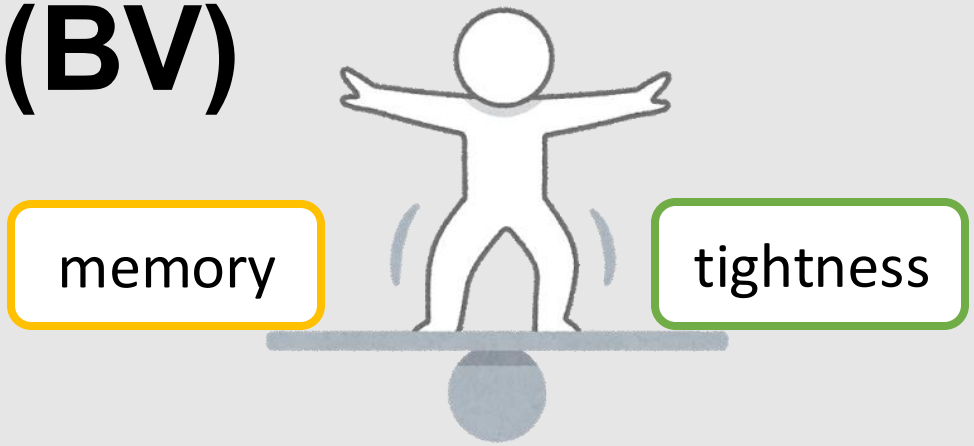


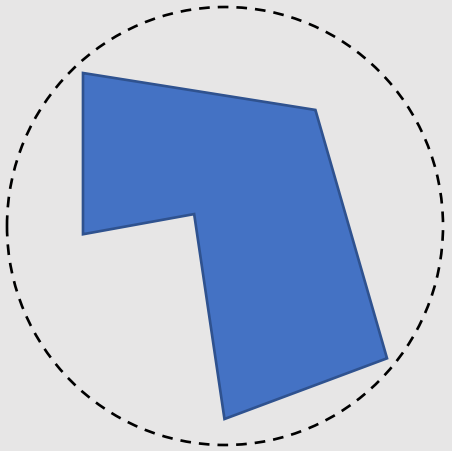
Broad Phase


Narrow Phase

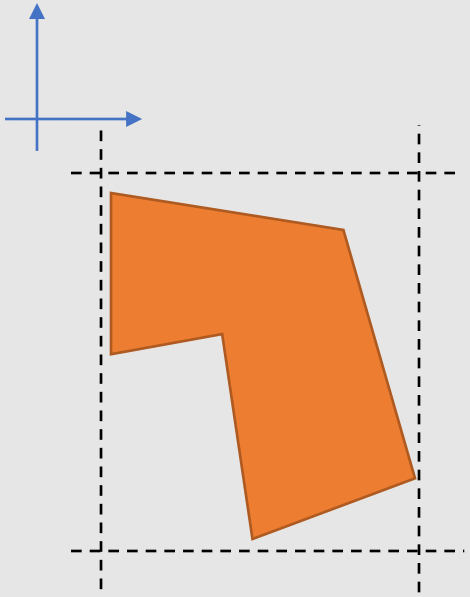
# Types of Bounding Volume (BV)

- Easy evaluation (convex shape!)
- Tightly fit to object's shape
- Low memory footprint

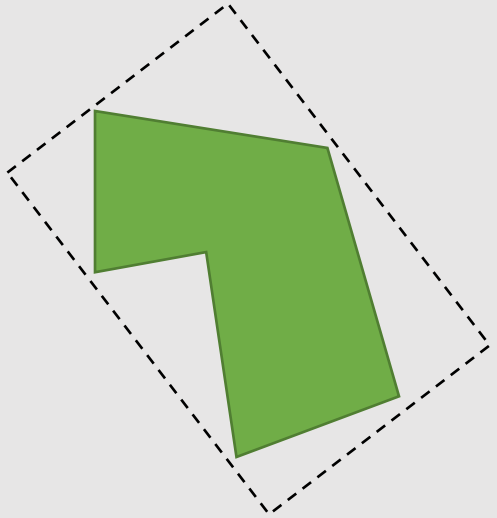


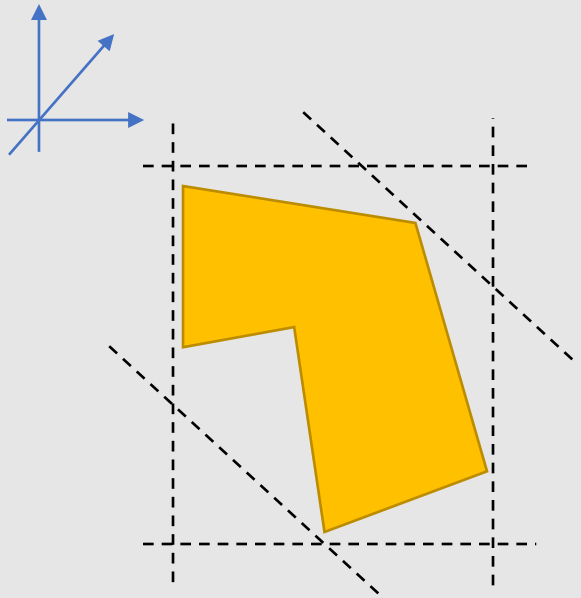
**Sphere**



**AABB**
Axis-Aligned Bounding Box



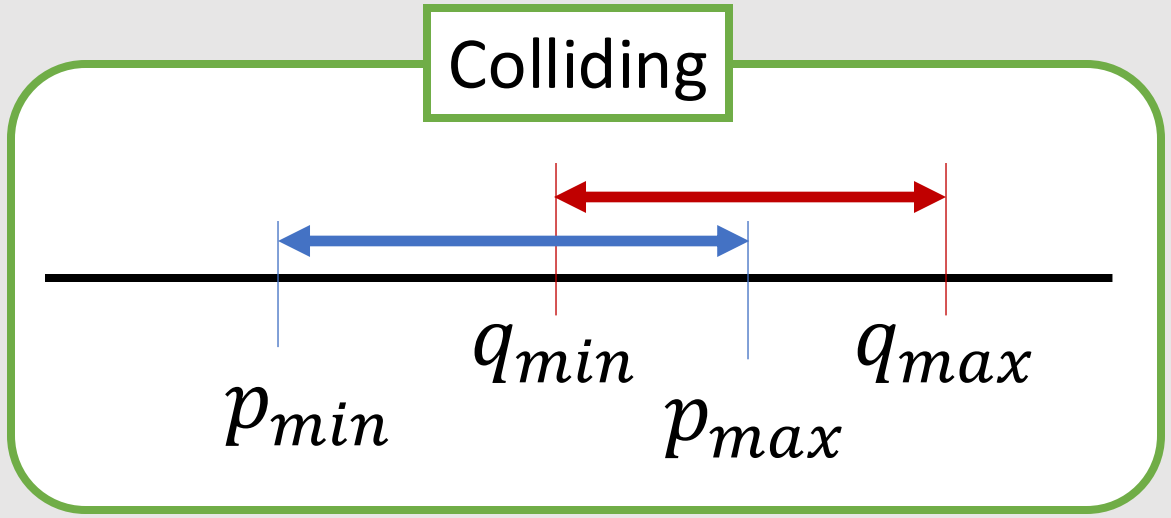
**OOBB**
Object-Oriented Bounding Box

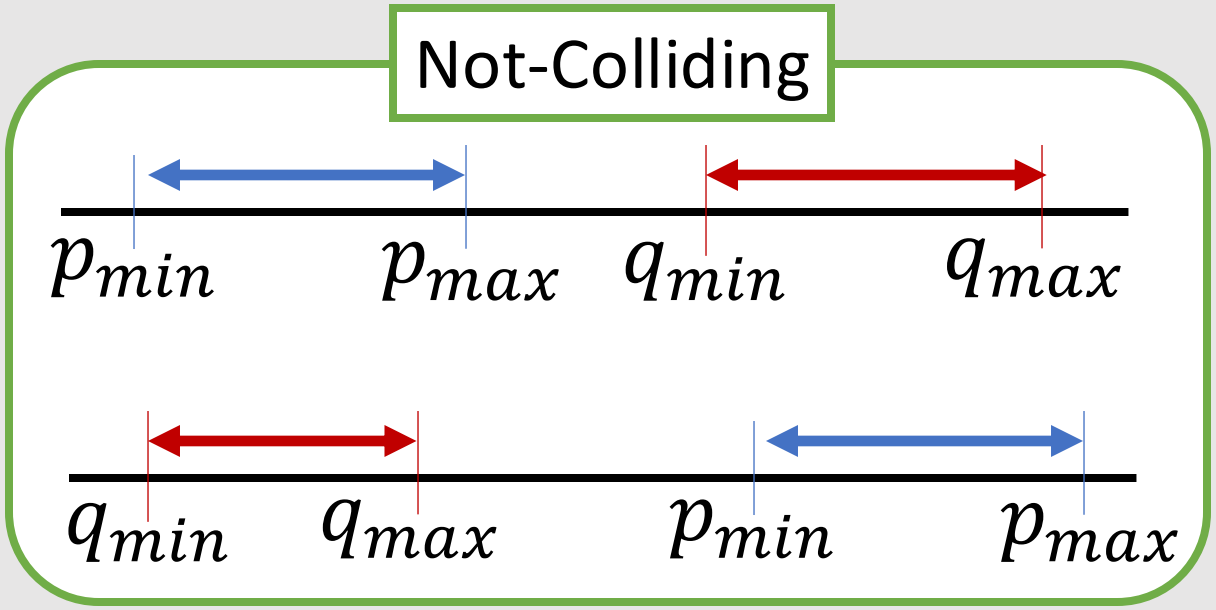


**k-DOP**
discrete orientation polytope

# 1D Collision Detection

- What is the condition that two line segments intersect?




$(p_{max} > q_{min})$ and $(q_{max} > p_{min})$

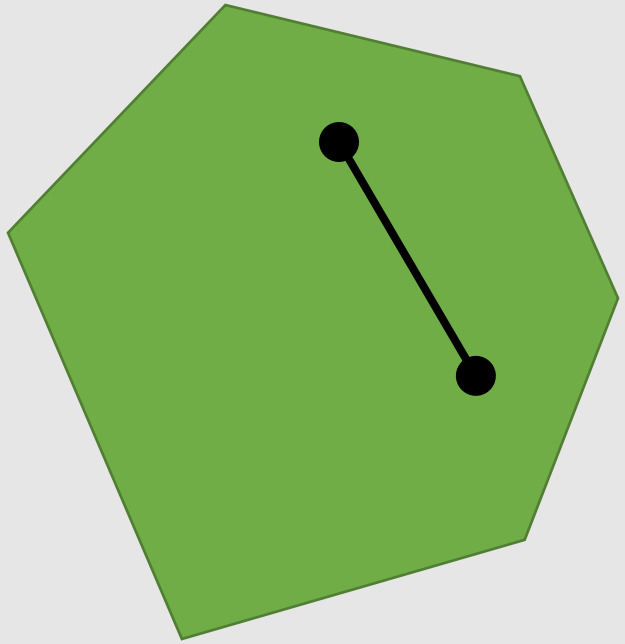
Logical inverse

$(p_{max} < q_{min})$ or $(q_{max} < p_{min})$

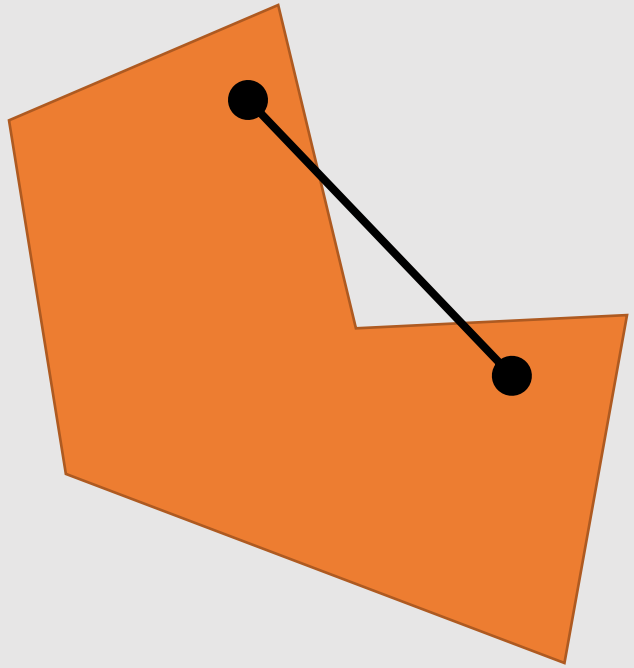
# What is “Convex” Shape

- Interpolation of two points is always included

Convex

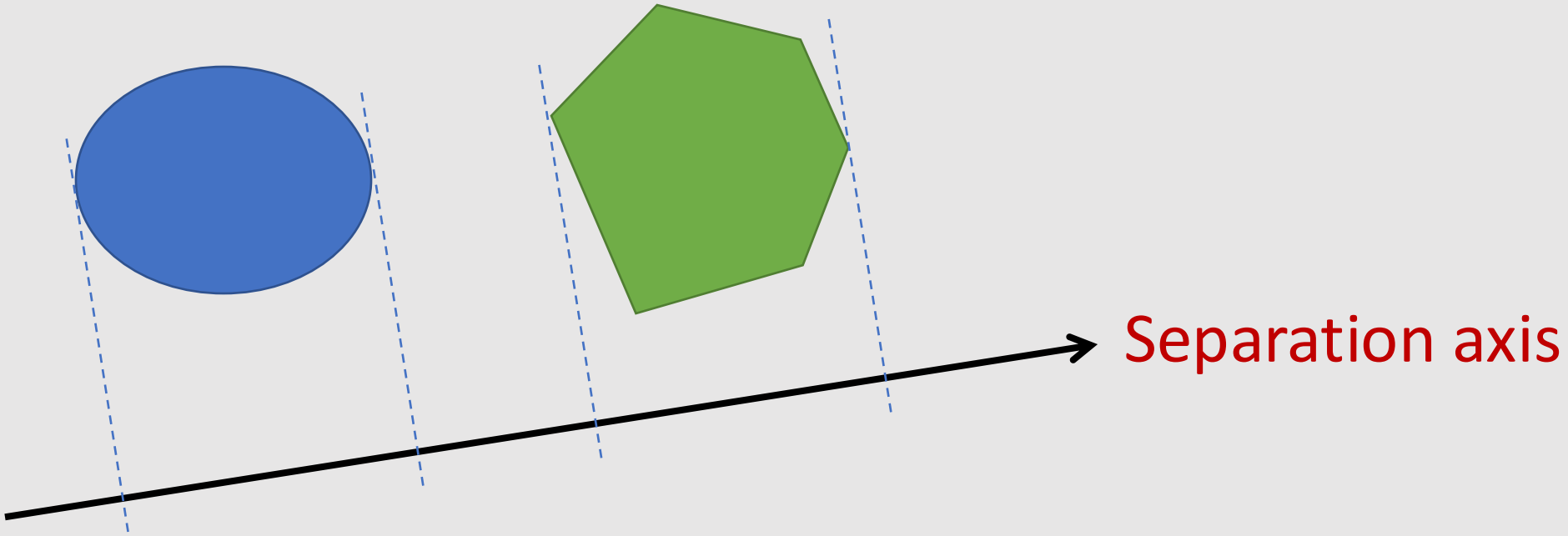


Non-Convex



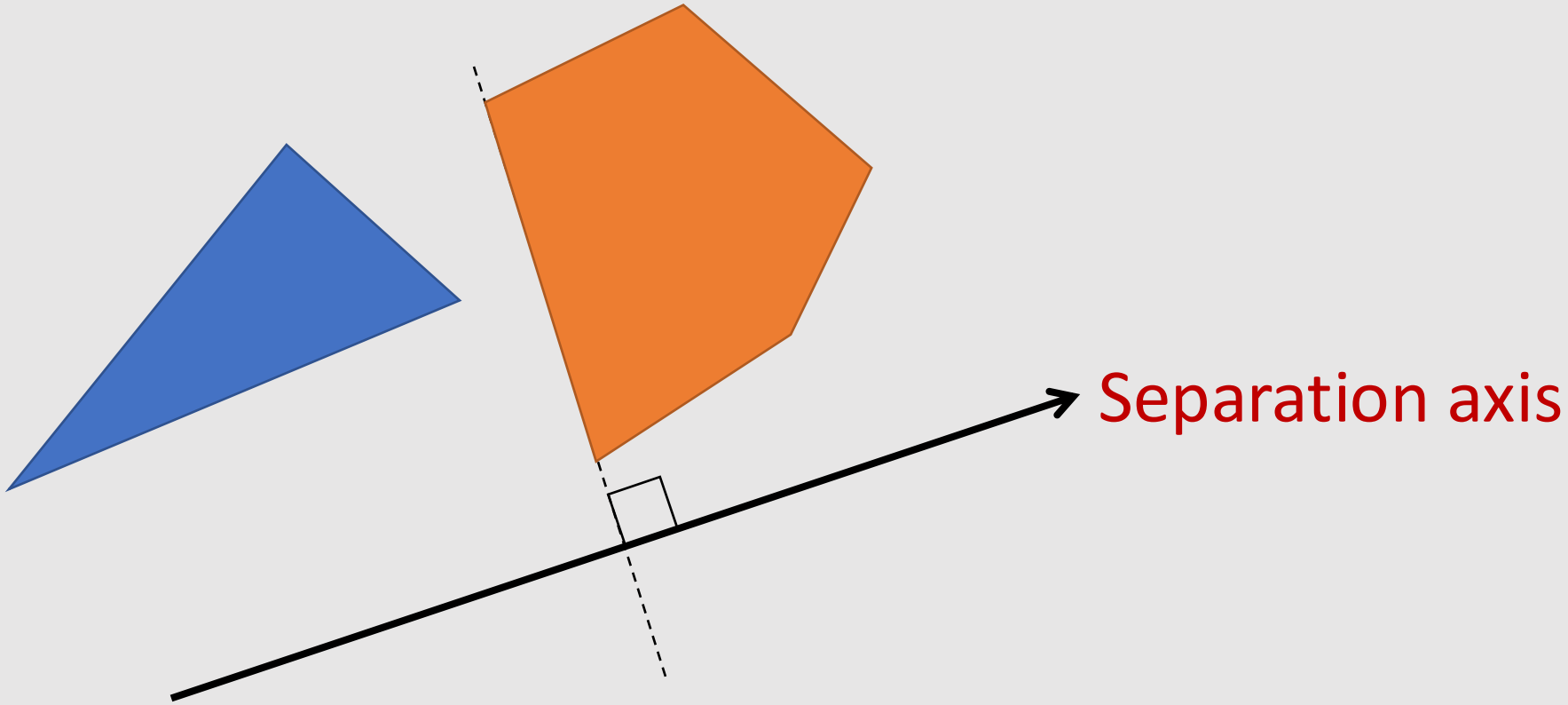
# Separation Axis Theorem (SAT)

- If two convex shapes do not collide, there exists an axis where their projections will not overlap



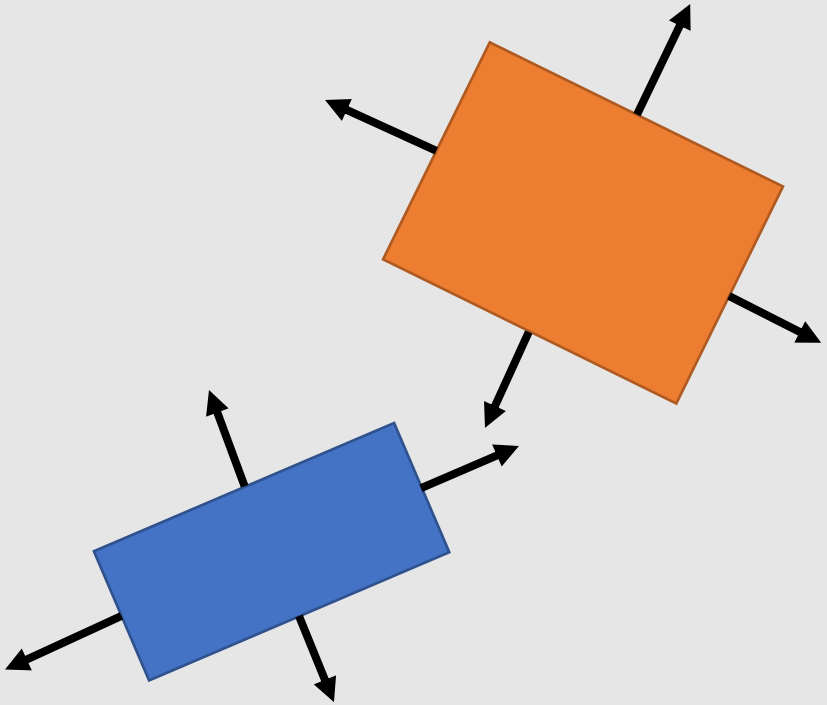
# Separation Axis Theorem for 2D Polygons

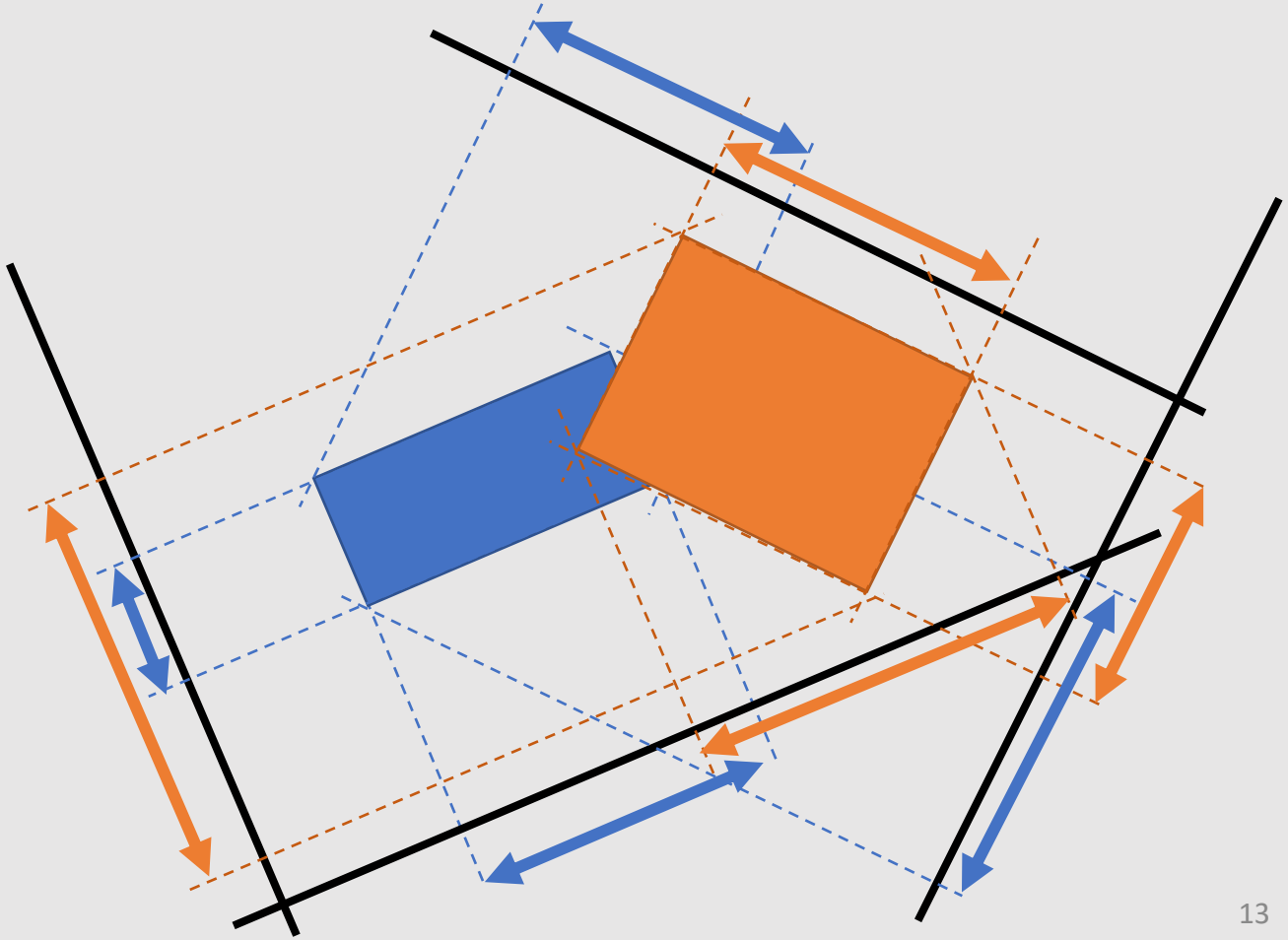
- One of the edges will be perpendicular to the separation axis



# Collision Detection for 2D Polygons

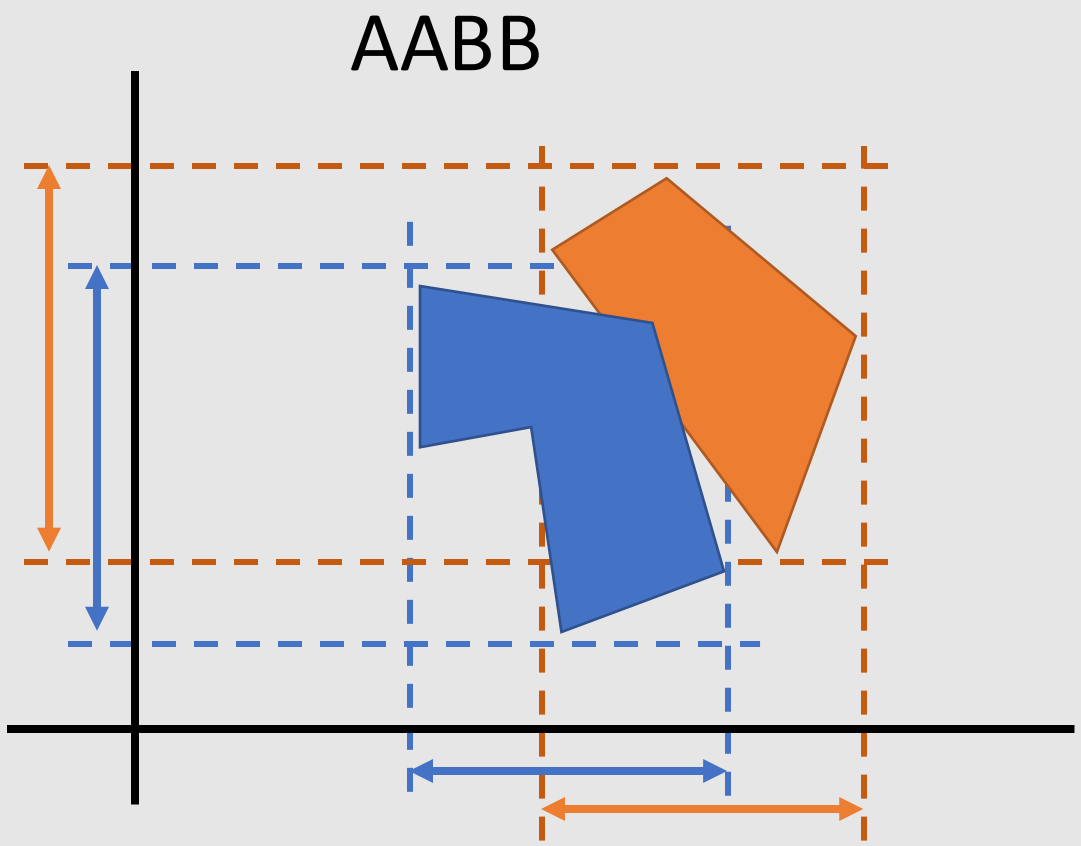
- Check all the axes perpendicular to polygon's edges

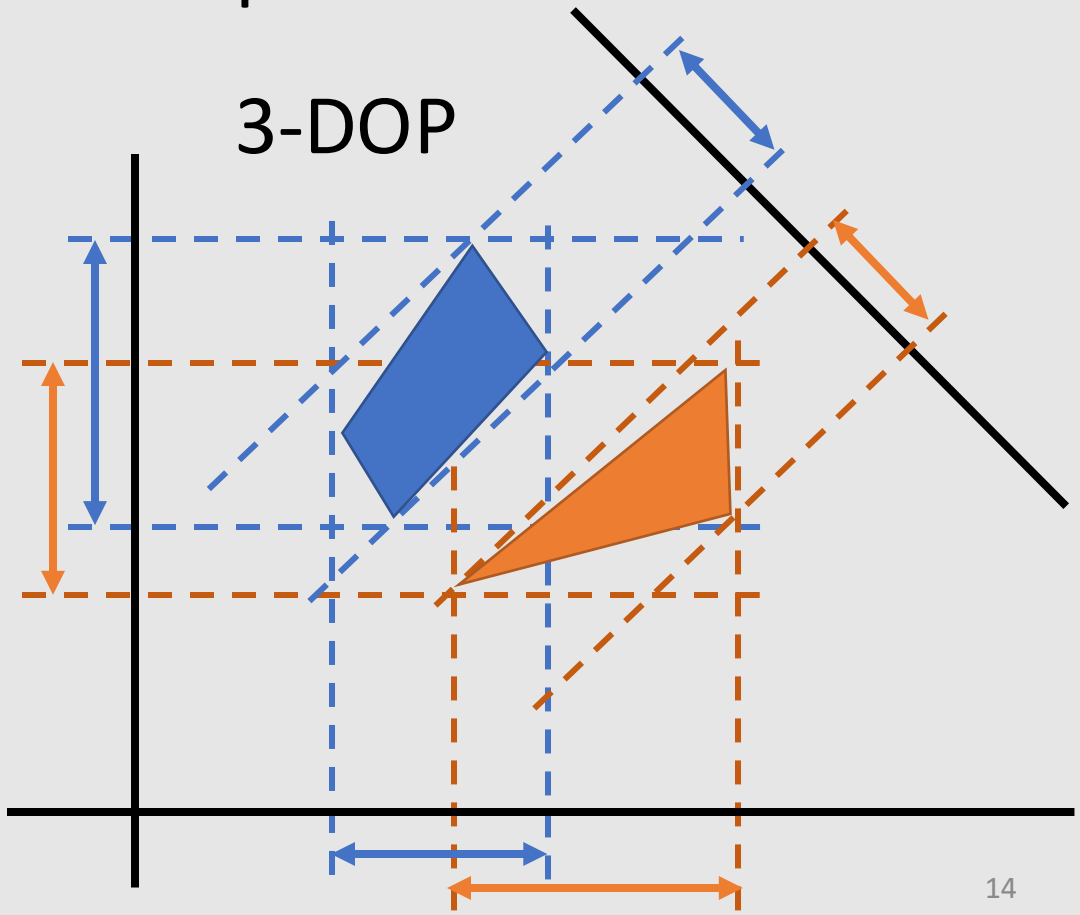




# Collision of AABB and k-DOP

- Project the Bounding Volume (BV) on axes
- Two BVs collide if all projections overlap



# Data Structure of AABB & k-DOP

- Minimum and maximum along the axis

```
template <int naxis>
class CKdops
{
public:
  double minmax[naxis][2];
};

constexpr double axes[3][2] = {
  {0,1},
  {1,0},
  {1,1} };
std::vector< CKdops<3> > aKdops;
```

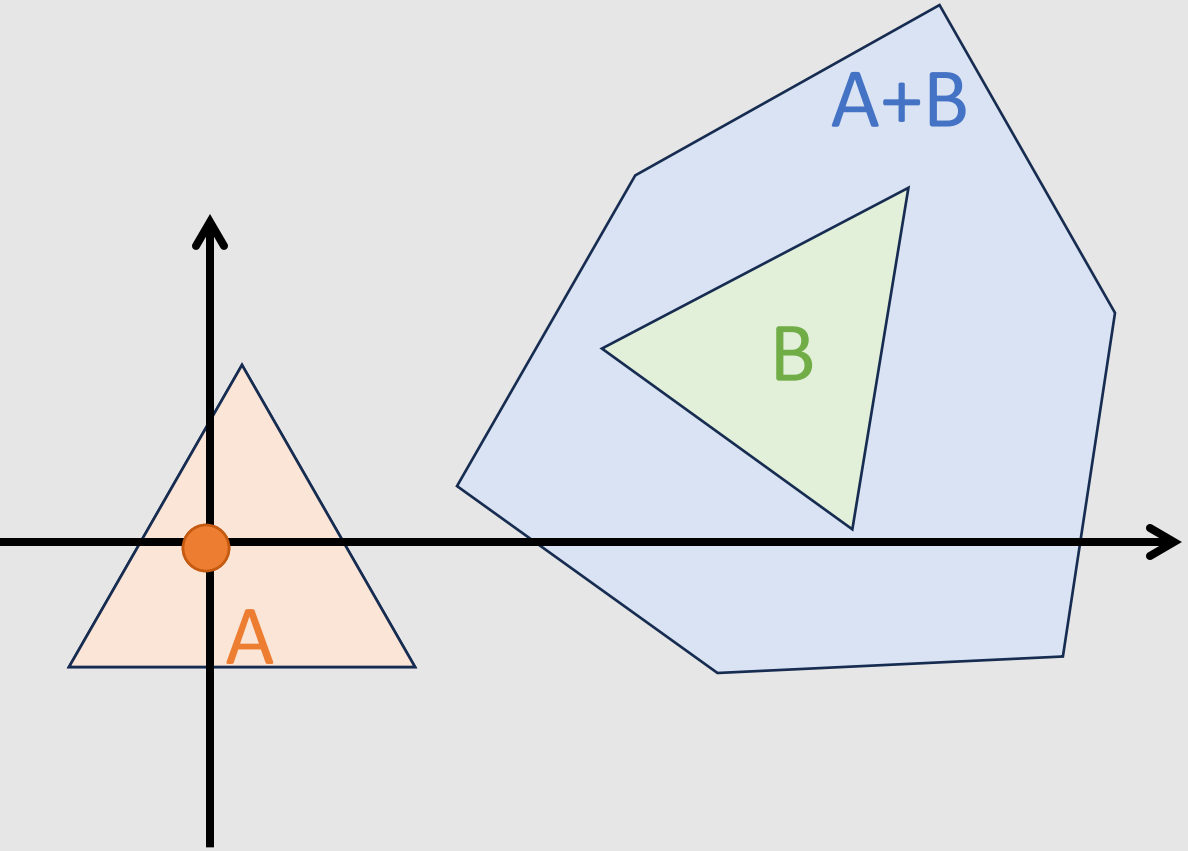
Non-type template parameter (compile time argument)

# GJK Algorithm

Interactive web page: https://cse442-17f.github.io/Gilbert-Johnson-Keerthi-Distance-Algorithm/

# Minkowski Addition

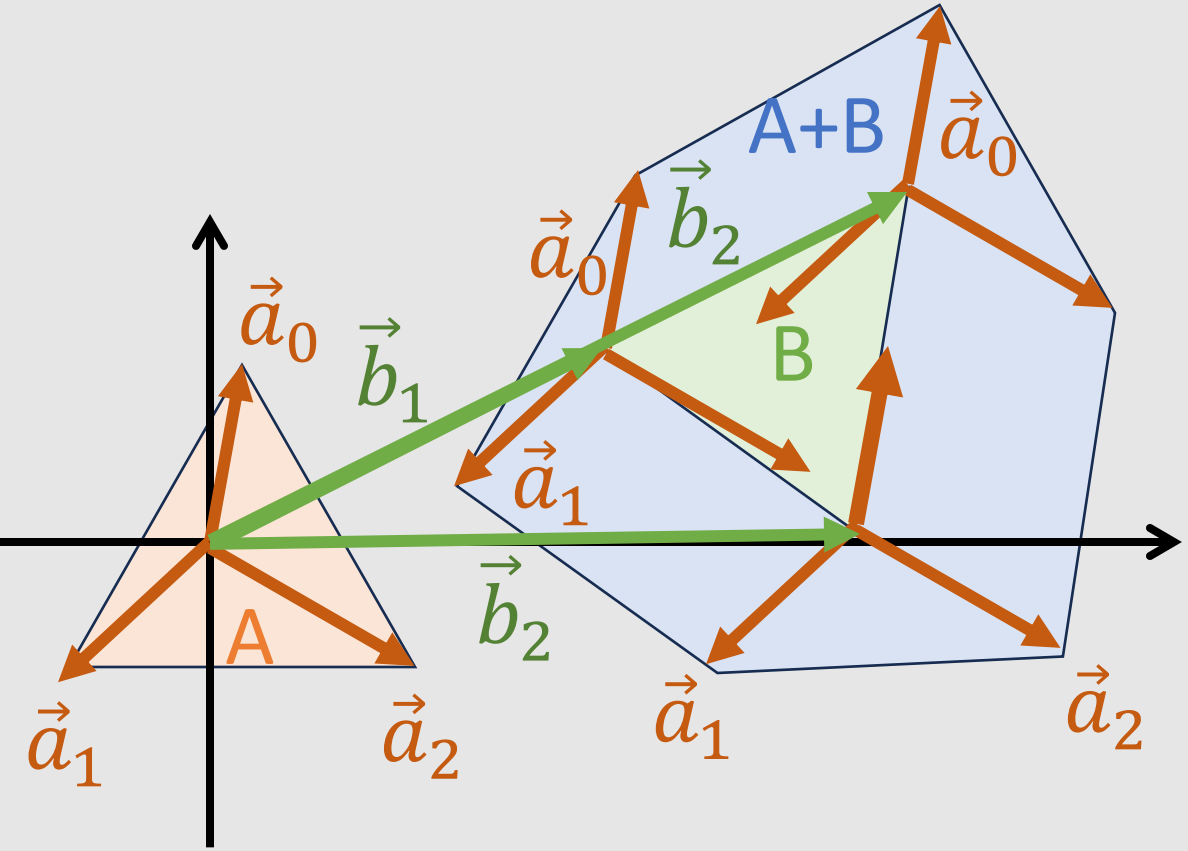
- A+B can be computed by moving A inside B




$$A + B = \{\vec{a} + \vec{b} | \vec{a} \in A, \vec{b} \in B\}$$

# Minkowski Addition for Convex Shapes

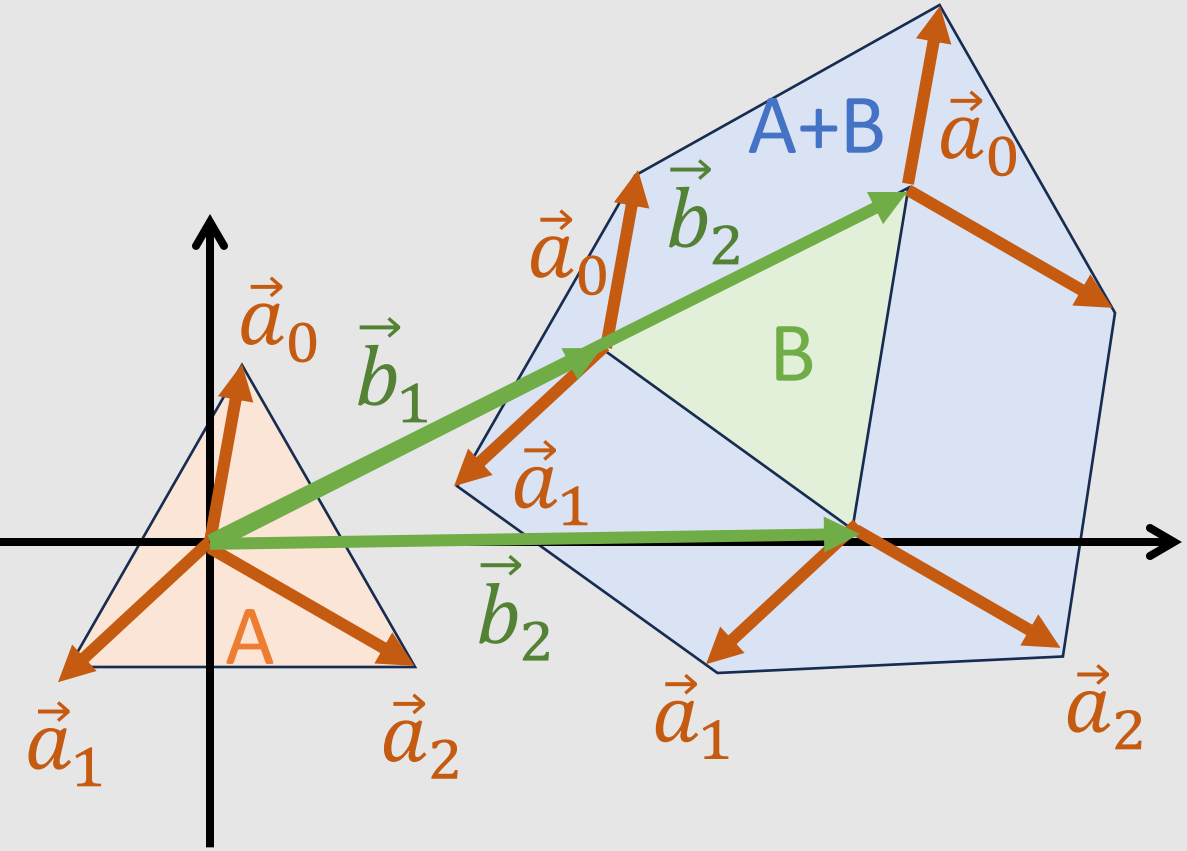
- Assume A and B are convex
- A+B is the convex hull of vertex coordinate addition



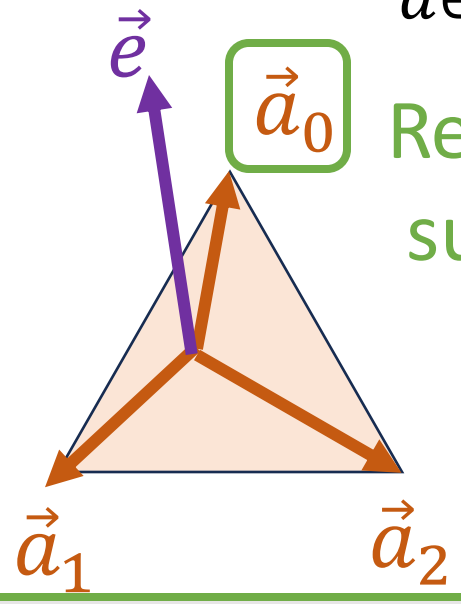

$$A + B = ConvexHull(\vec{a}_i + \vec{b}_j)$$

# Support Function and Minkowski Addition

- Given a direction $\vec{e}$, support function returns the furthest point
- Support function directly gives the vertices of the convex hull



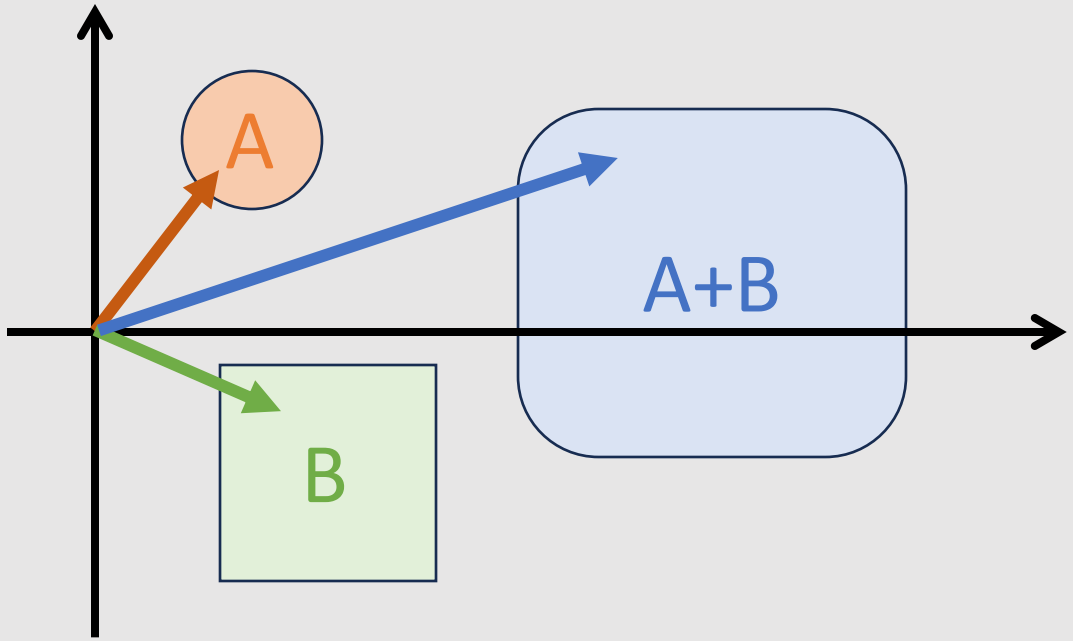

$$\text{Support}_{\text{A}}(\vec{e}) = \arg \max_{\vec{a} \in \{\vec{a}_0, \vec{a}_1, \vec{a}_2\}} (\vec{e} \cdot \vec{a})$$



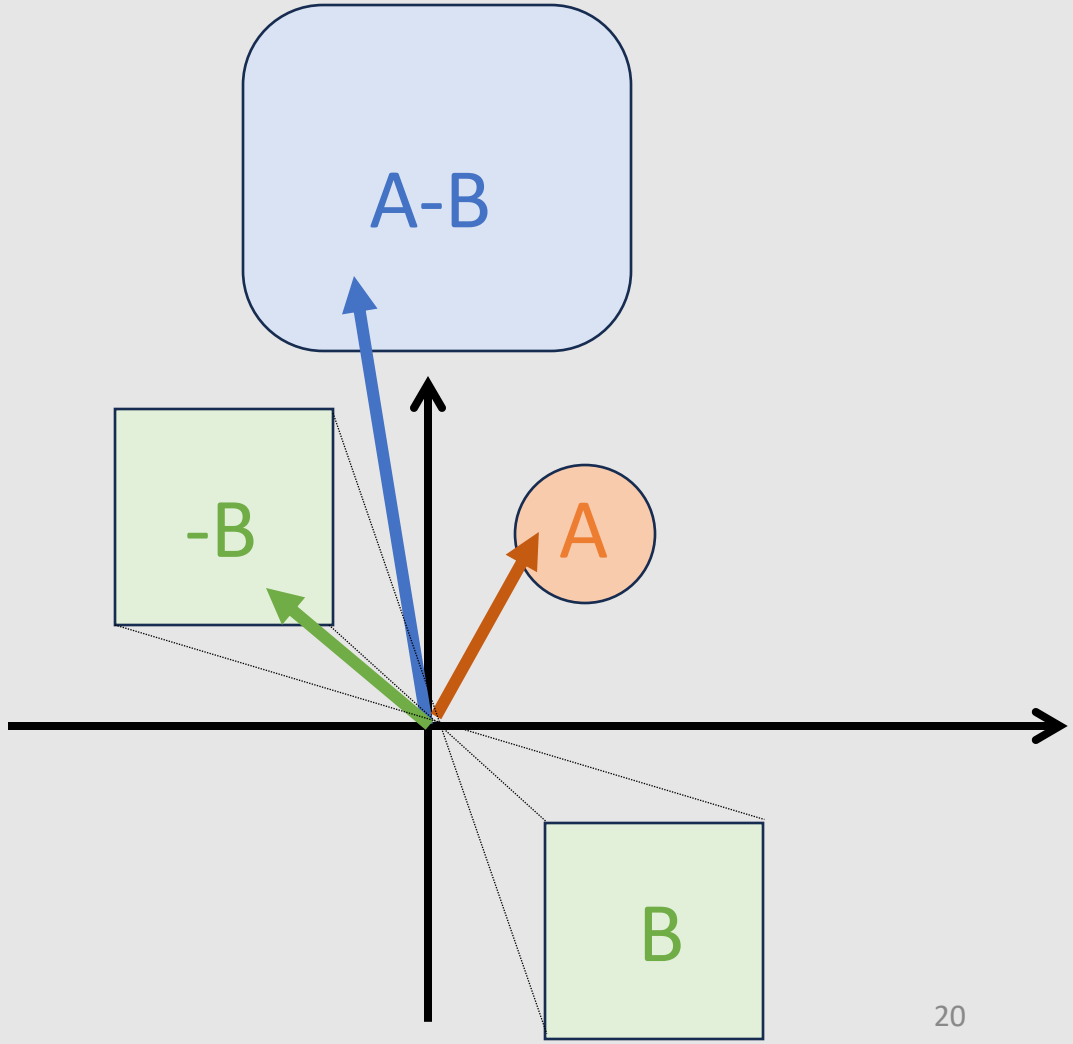

Result of support

# Minkowski Substraction

$$A + B = \{\vec{a} + \vec{b} | \vec{a} \in A, \vec{b} \in B\}$$



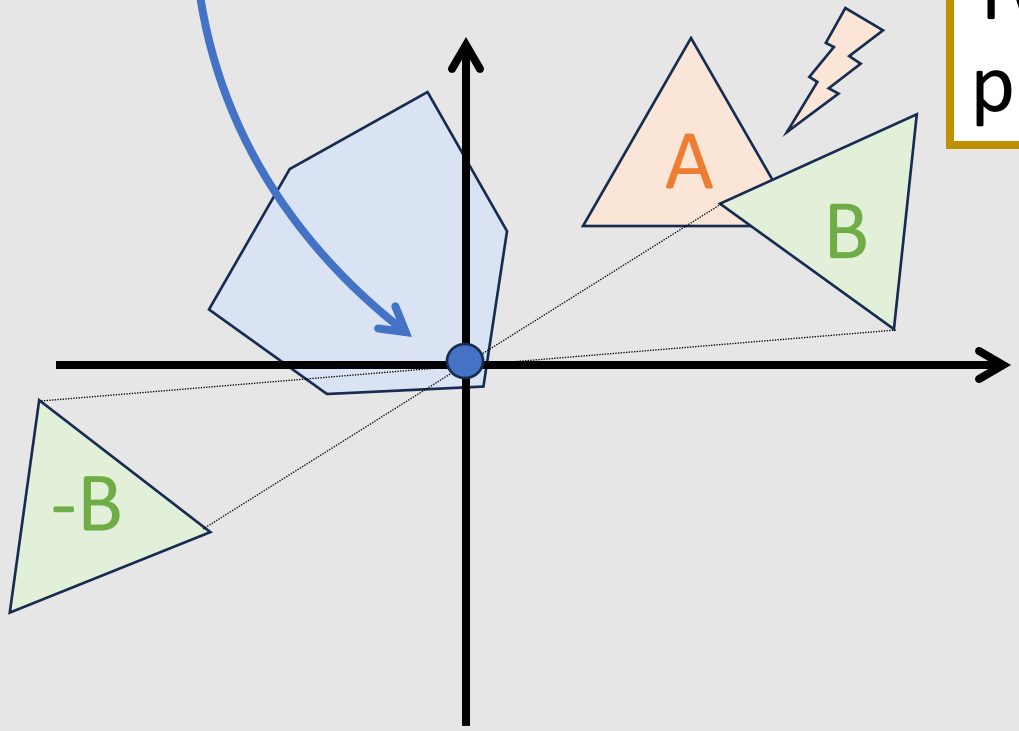

$$A - B = \{\vec{a} - \vec{b} | \vec{a} \in A, \vec{b} \in B\}$$



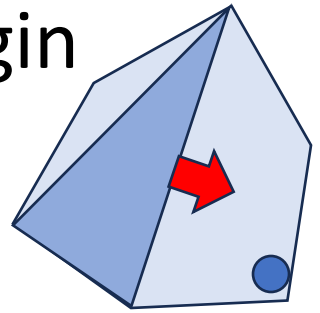
# Minkowski Subtraction & Collision

Two convex polygon's Minkowski subtraction contains origin

Two convex polygons collide




GJK algorithm itegratively update a simplex such that it contains the origin

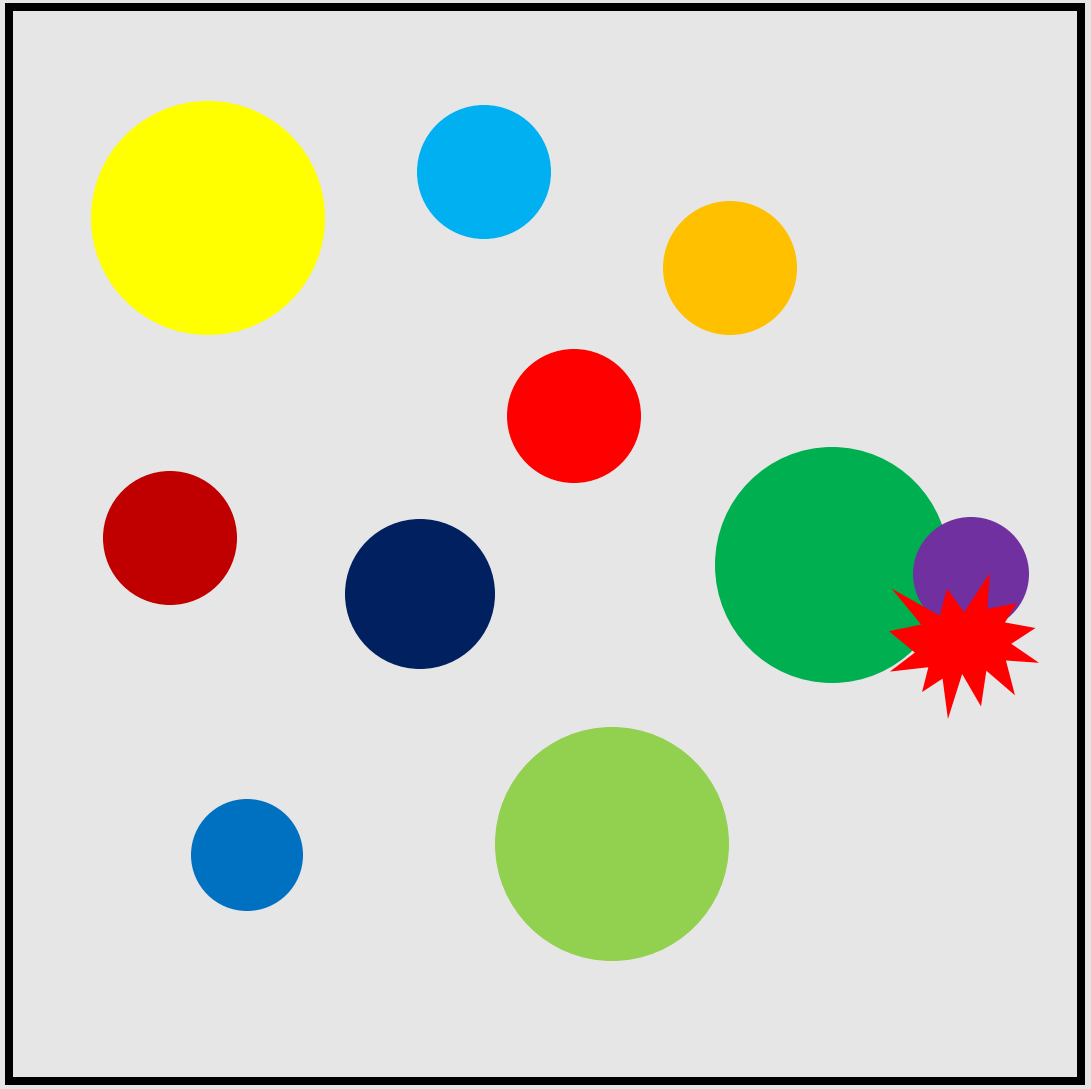


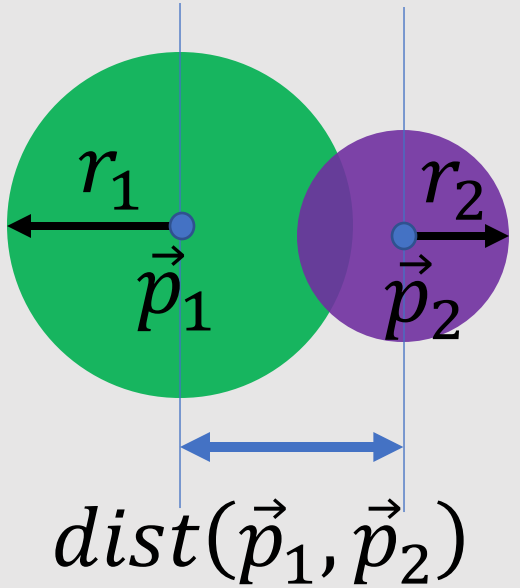
# **GJK Algorithm** (YouTube Video by Reducible)

**Video by Reducible**
**A Strange But Elegant Approach to a Surprisingly Hard Problem (GJK Algorithm)**
**https://www.youtube.com/watch?v=ajv46BSqcK4**

# Broad-phase Collision Detection

# How We can Find Collisions of Circles?






$dist(\vec{p}_1, \vec{p}_2) \leq r_1 + r_2$ ⇒Collision

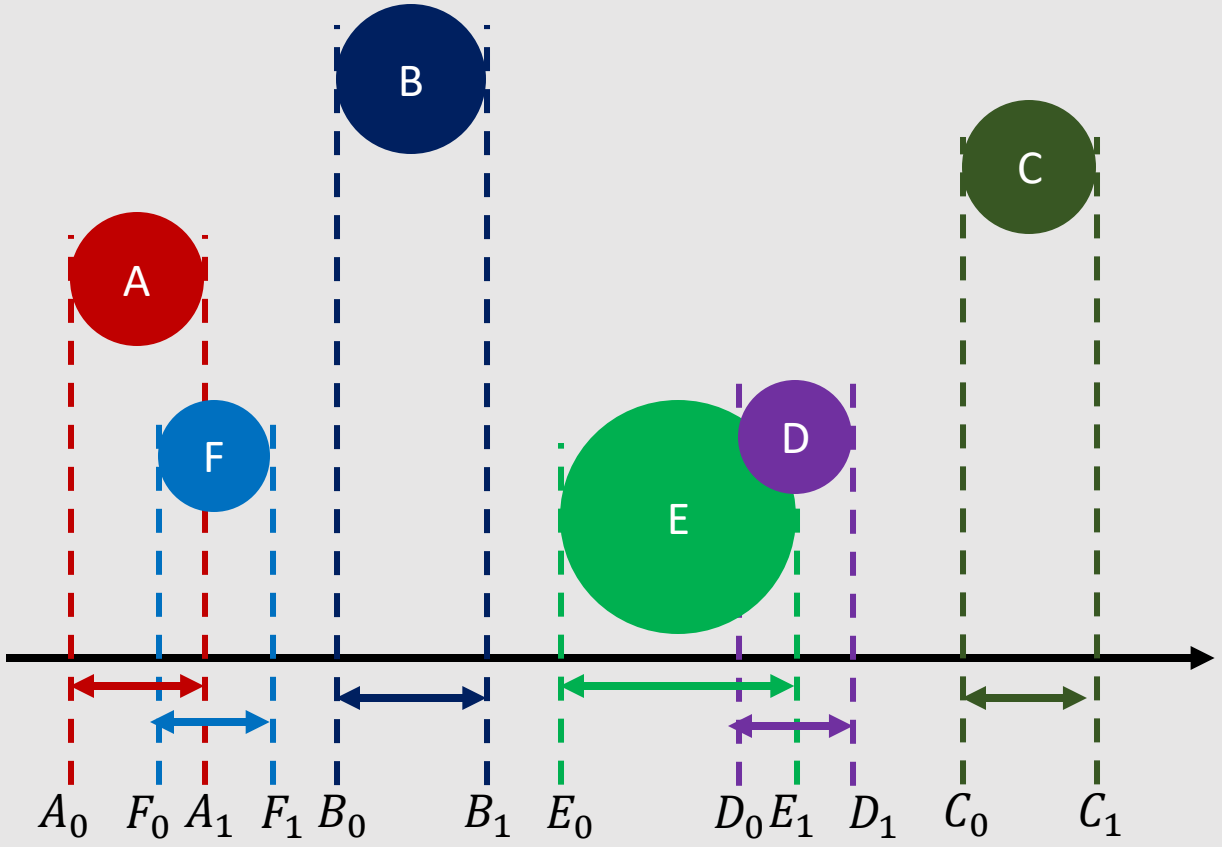
# Approaches

- ~~Brute force approach~~
- Sweep & Prune method
- Spatial Hashing (e.g., Regular grid)
- Spatial Partitioning (e.g., KD-tree)
- Bounding Volume Hierarchy (BVH)

We four are awesome!

# Sweep & Prune (Sort & Sweep) Method

- Simple but effective culling method




$$\{A_0, A_1, B_0, B_1, C_0, C_1, D_0, D_1, E_0, E_1, F_0, F_1\}$$

sort

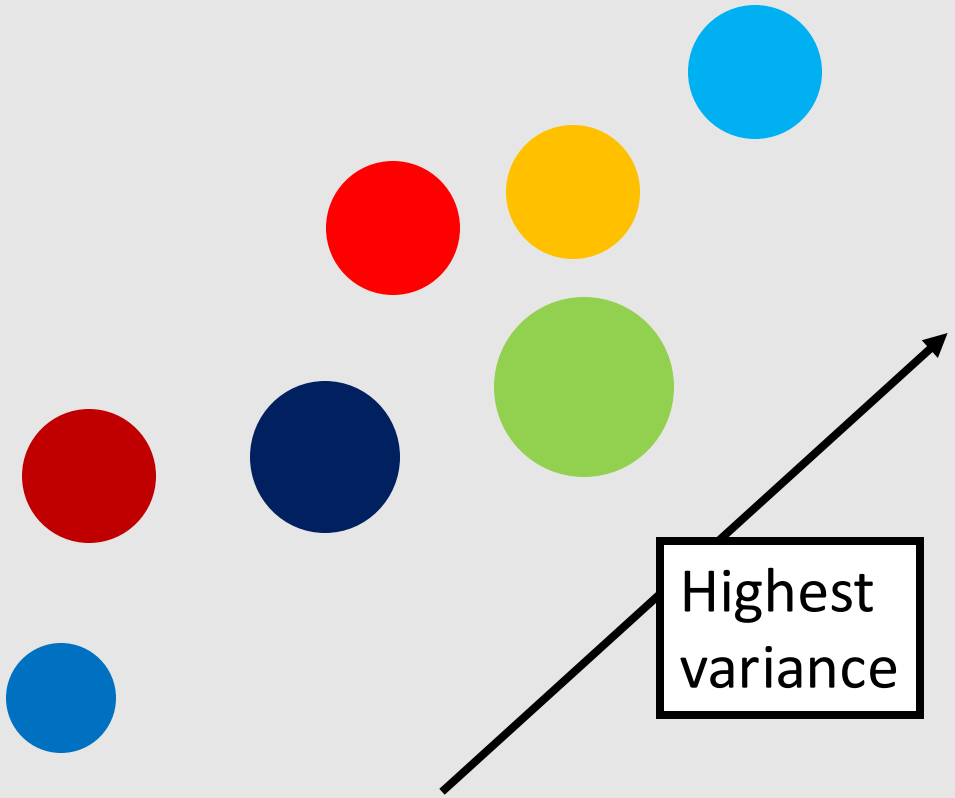
$$\{A_0, F_0, A_1, F_1, B_0, F_1, E_0, D_0, E_1, D_1, C_0, C_1\}$$

$X_0$: put X in the set

$X_1$: remove X in the set

# How to Choose Sweeping Axis ?

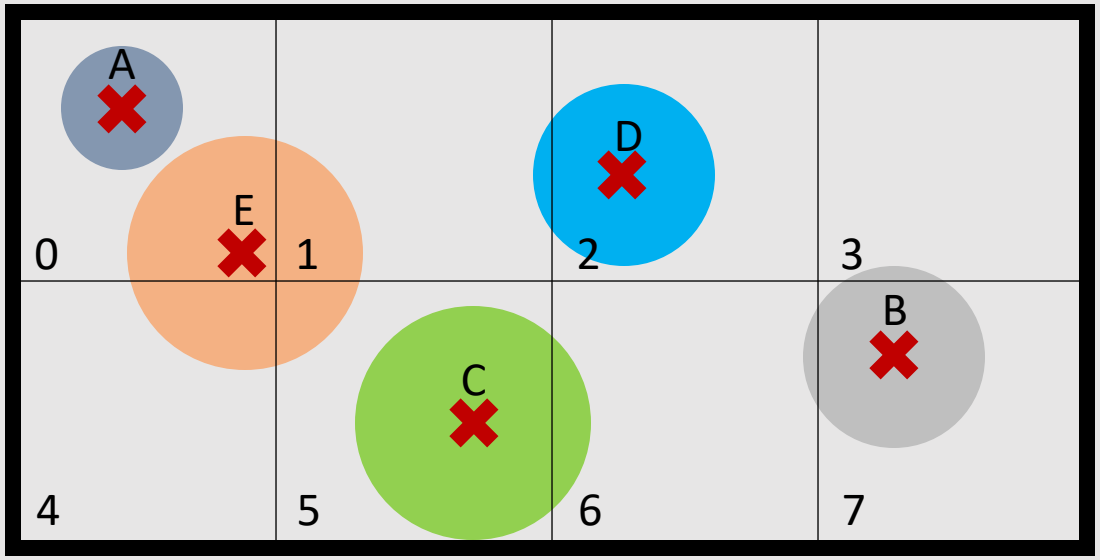
- kDOPs -> Sweep in the kDOPs' axis
- Sphere, AABB, OOBB -> XYZ-axis or PCA



# Spatial Hashing using Regular Grid

- Putting circles in a grid based on circles' center positions
- Grid length is maximum diameter of the circle

  ➡ Look only 1-ring neighborhood



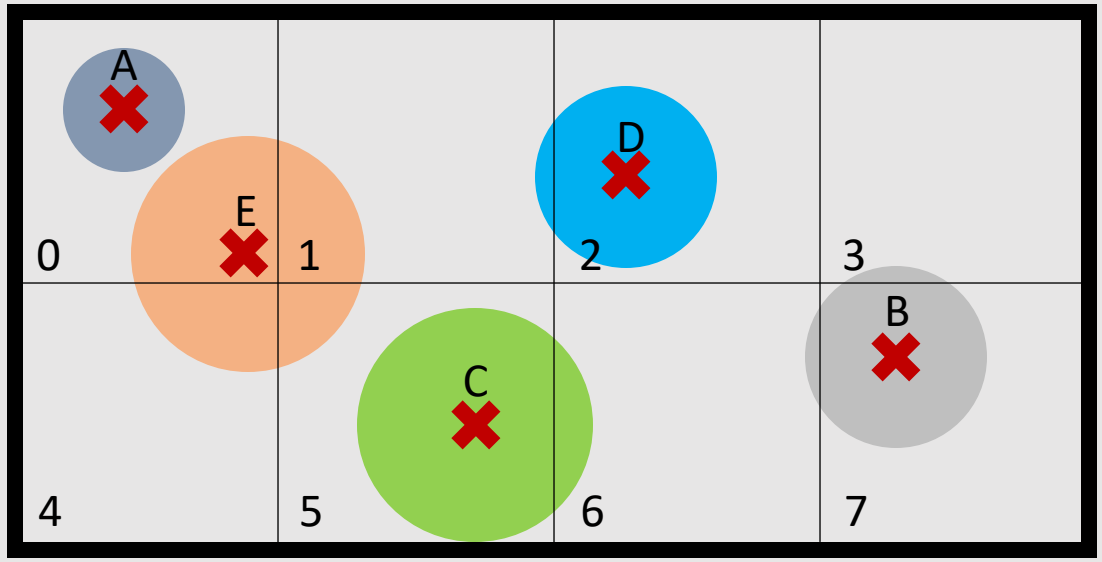

Possible collisions:

{A,E}, {E,C}, {C,D}, {D,B}

No need to check for {E,D},{C,B}…etc

# Spatial Hashing using Regular Grid

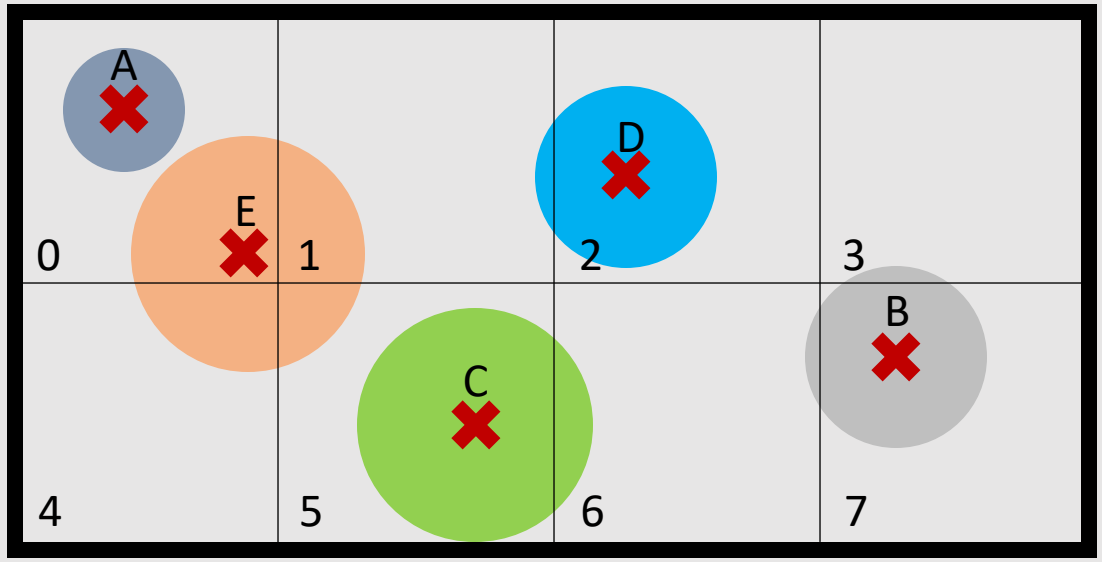
- Creating look-up table from grid index to circle index




| circle index | A | B | C | D | E |
|---|---|---|---|---|---|
| grid index | 0 | 7 | 5 | 2 | 0 |

# Spatial Hashing using Regular Grid

- Creating look-up table from grid index to circle index




| circle index | A | B | C | D | E |
|---|---|---|---|---|---|
| grid index | 0 | 7 | 5 | 2 | 0 |

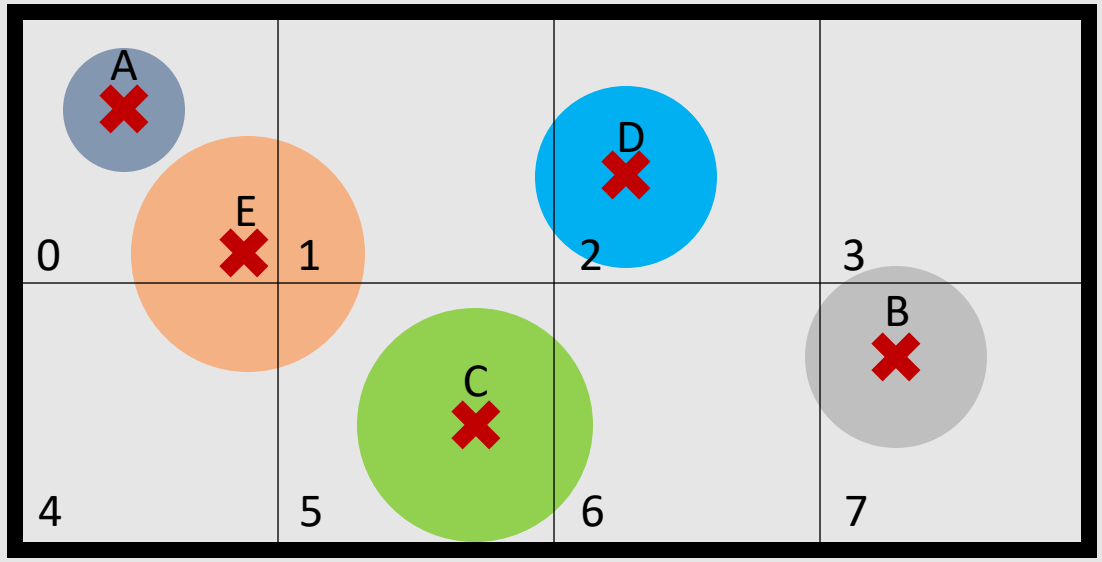
sort by the grid index

A=

| circle index | A | E | D | C | B |
|---|---|---|---|---|---|
| grid index | 0 | 0 | 2 | 5 | 7 |

# Spatial Hashing using Regular Grid

- Creating look-up table from grid index to circle index




| circle index | A | B | C | D | E |
|---|---|---|---|---|---|
| grid index | 0 | 7 | 5 | 2 | 0 |

sort by the grid index

A=

| circle index | A | E | D | C | B | |
|---|---|---|---|---|---|---|
| grid index | 0 | 0 | 2 | 5 | 7 | |

B=

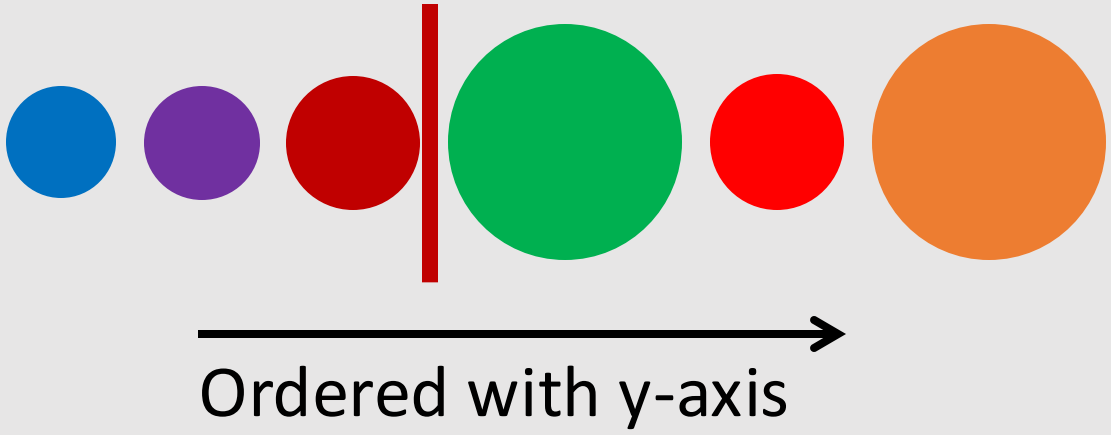
| index of A | 0 | 2 | 2 | 3 | 3 | 3 | 4 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|

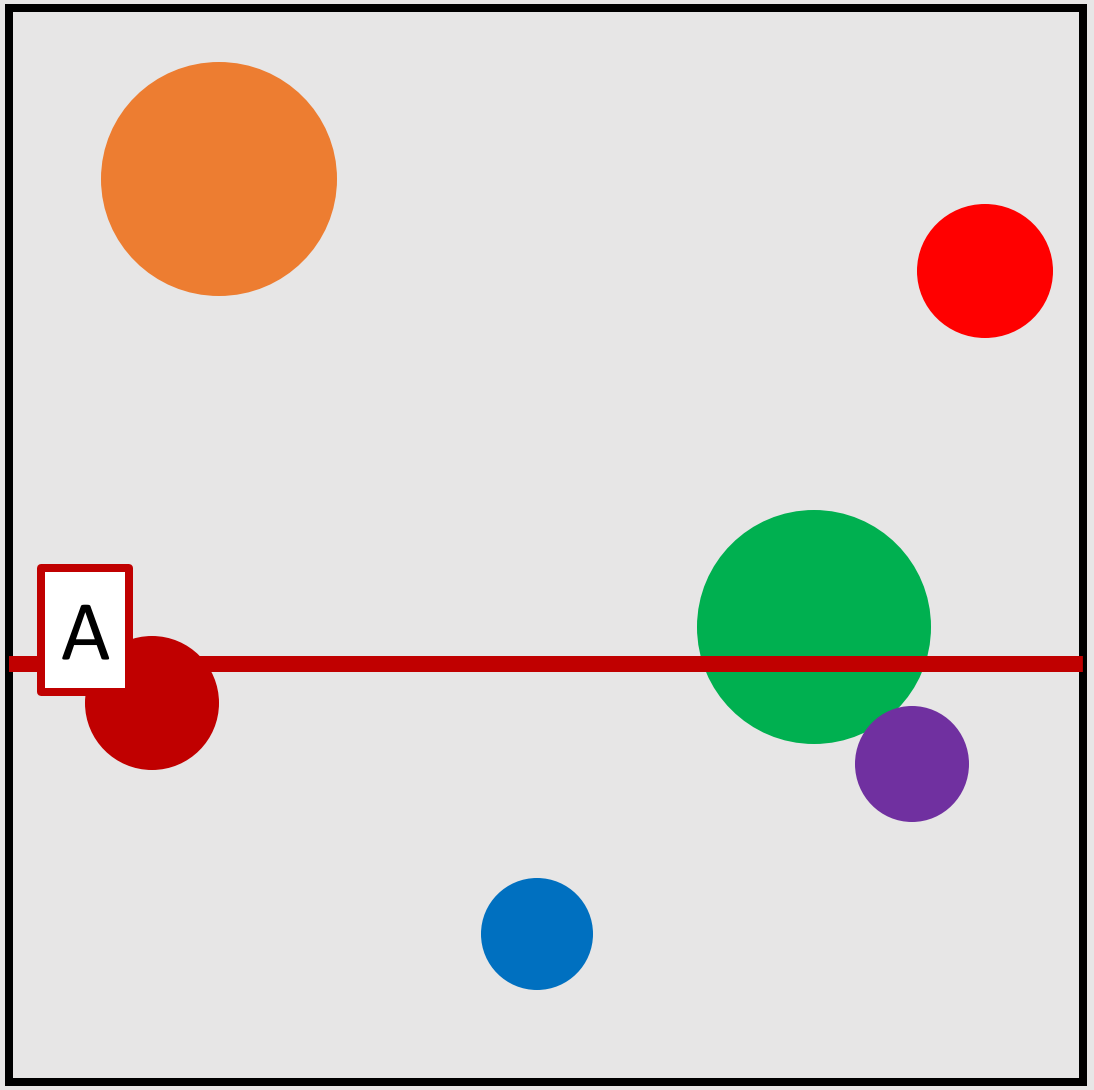
jagged array

B[igrid] <= j < B[igrid+1]
icircle=A[j]

# Space Partitioning with K-D Tree

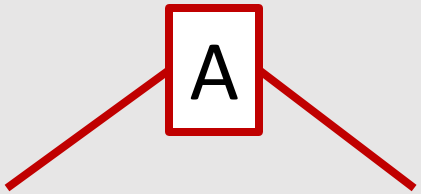
1. Select axis (e.g., y-axis)
2. Split the space along median

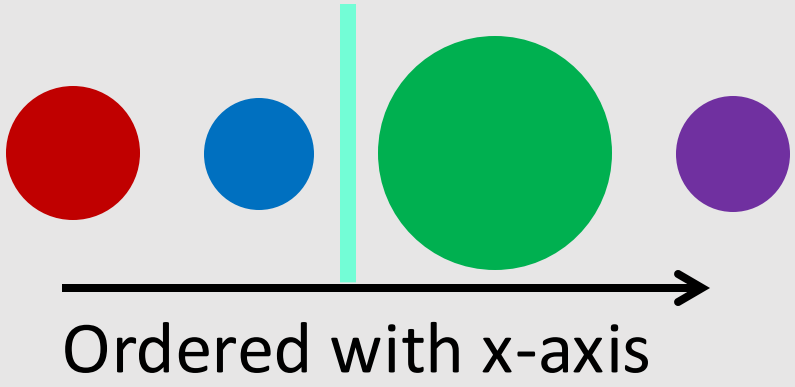


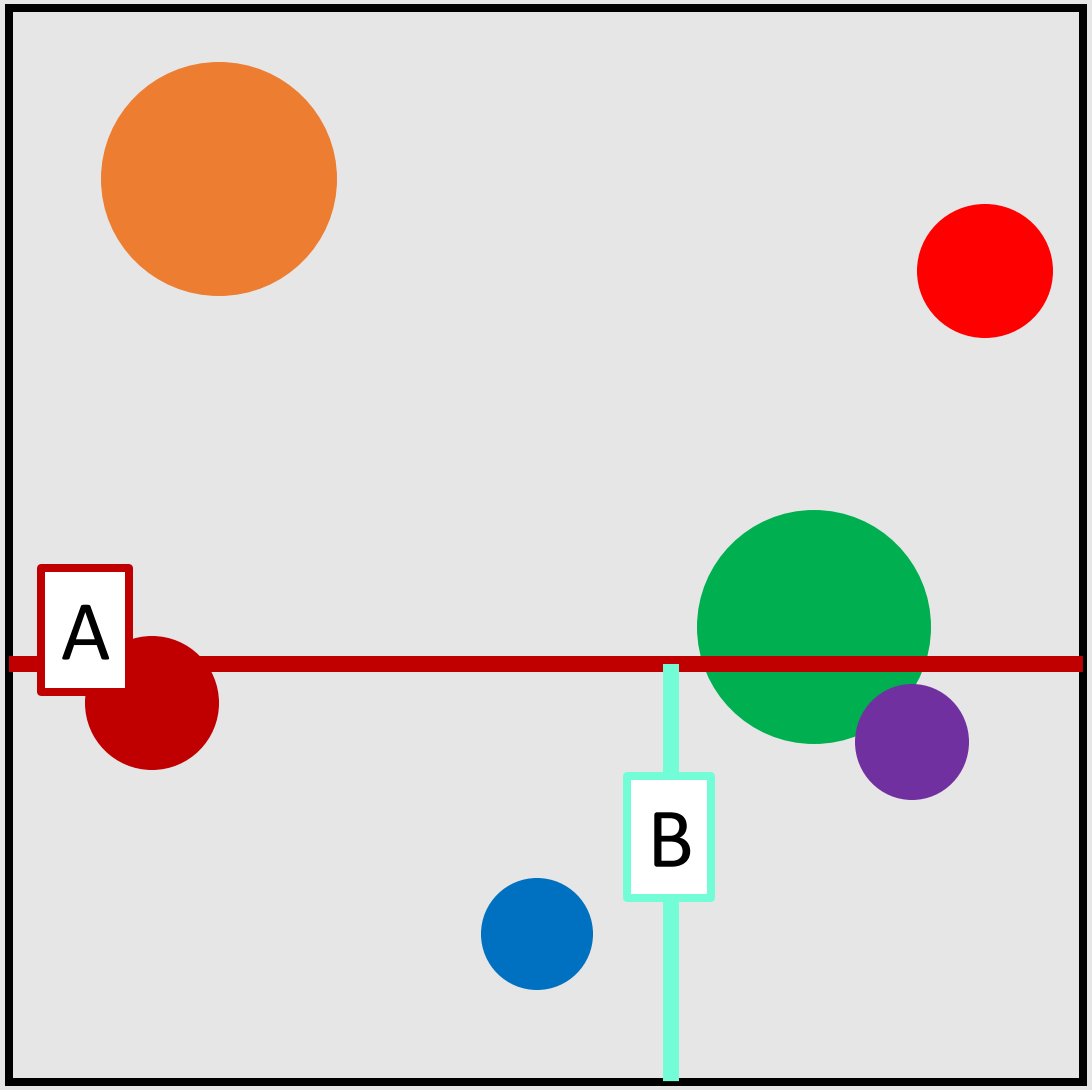



# Space Partitioning with K-D Tree

1. Select axis (e.g., y-axis)
2. Split the space along median
3. Repeat along other axis (e.g., x-axis)



# Space Partitioning with K-D Tree

1. Select axis (e.g., y-axis)
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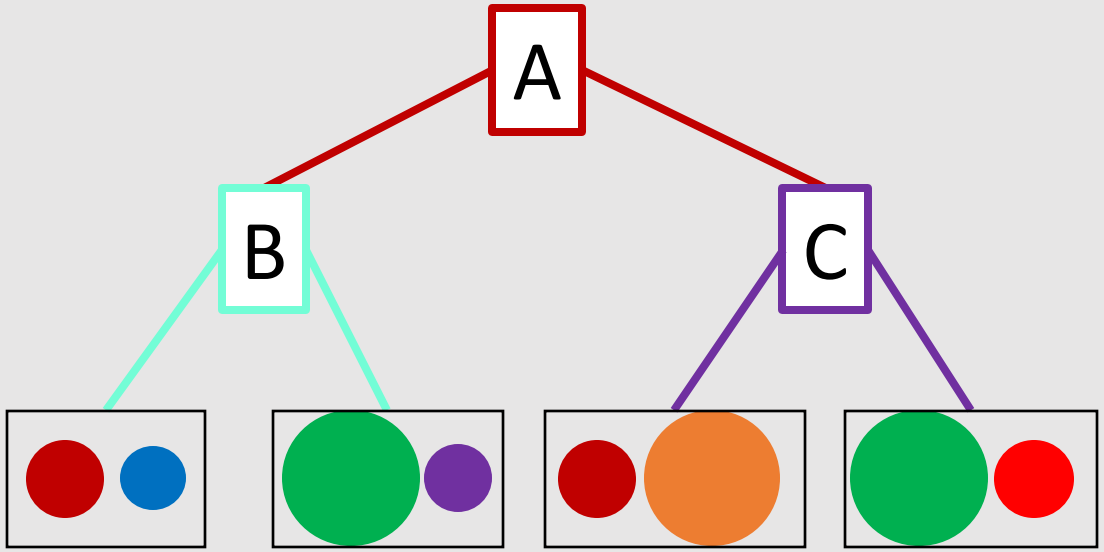
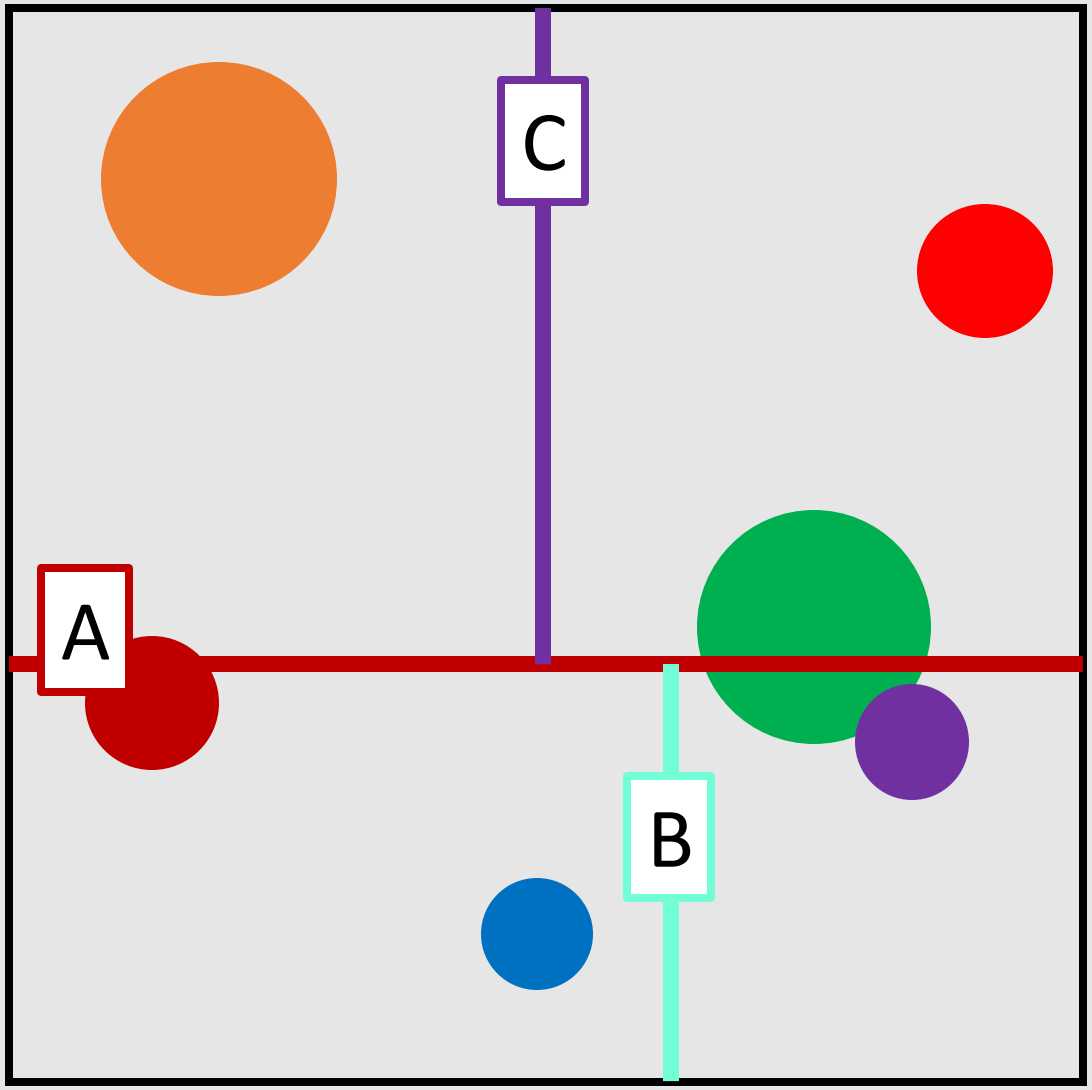
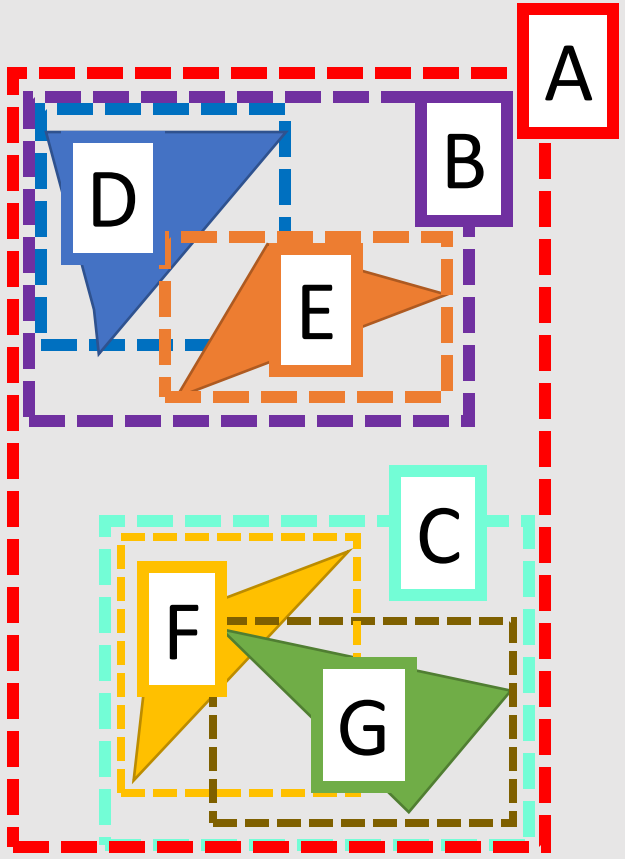
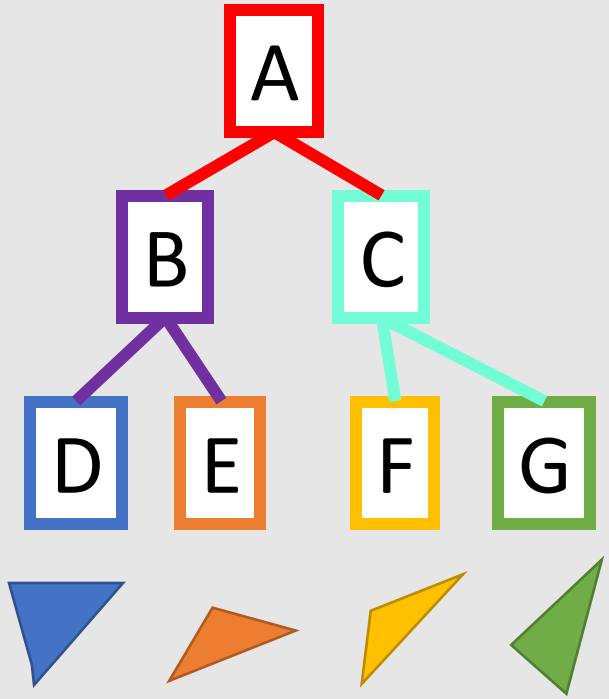


# Bounding Volume Hierarchy (BVH)

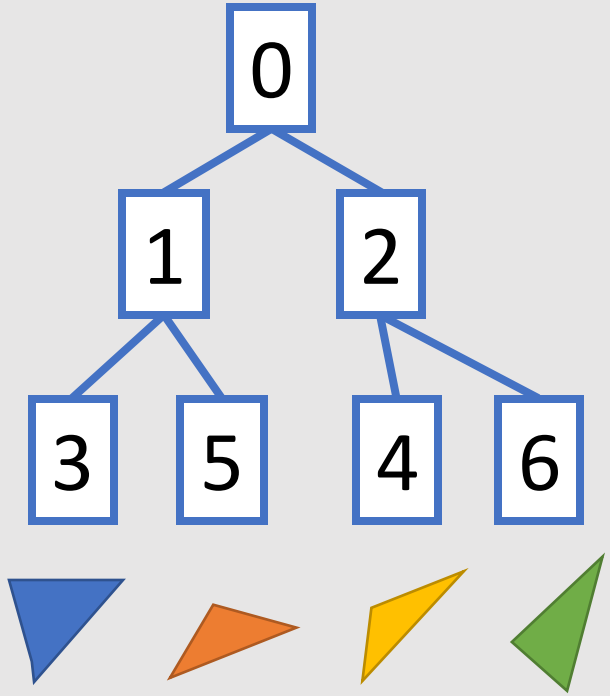
- Near triangles are in the same branch
- Each node has a BV that includes two child BVs



# Example of BVH Data Structure in C++

| index | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| left-child index | 1 | 3 | 4 | tri index | tri index | tri index | tri index |
| Right-child index | 2 | 5 | 6 | -1 | -1 | -1 | -1 |
| BV data | … | … | … | … | … | … | … |

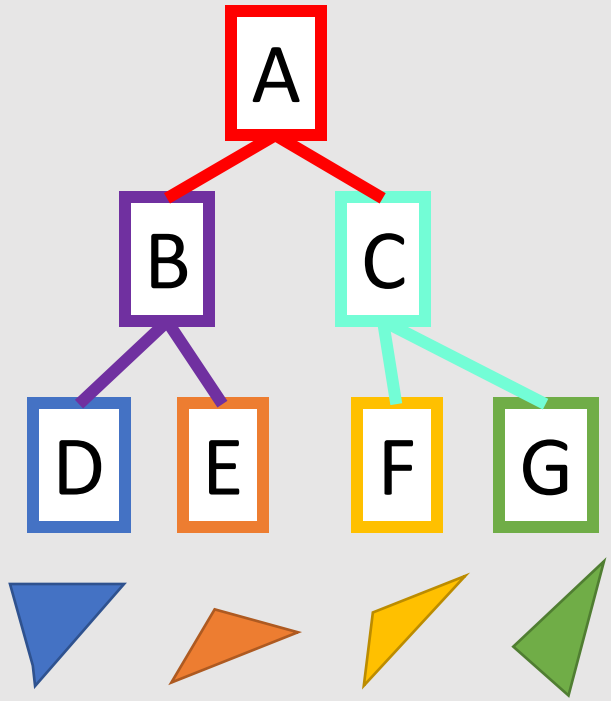



```
template <class T>
class CNodeBVH {
        unsigned int ichild_left;
        unsigned int ichild_right;
        T BV;
};

std::vector<CNodeBVH<CAABB>> aNodeBVH;
```

# Evaluation of BVH using Recursion

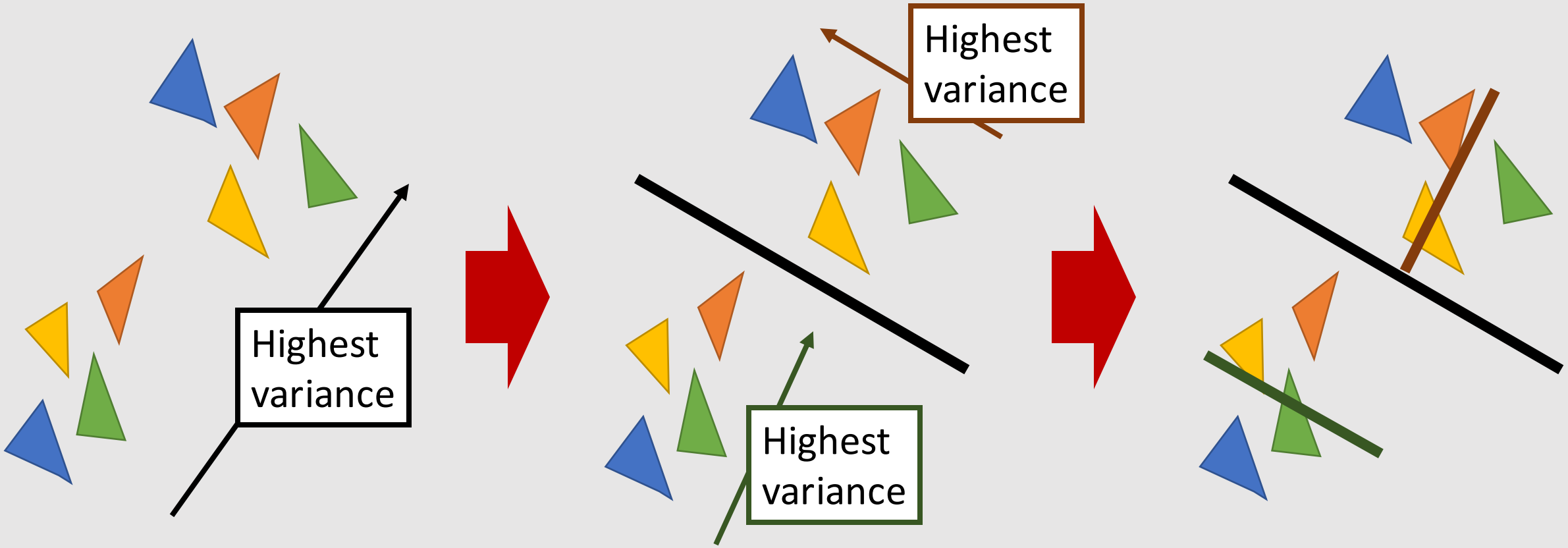
- Ask question to the root node -> if true the node asks the same question to two child nodes and so on




A, do you intersect with a ray?
A, do you have self-intersection?

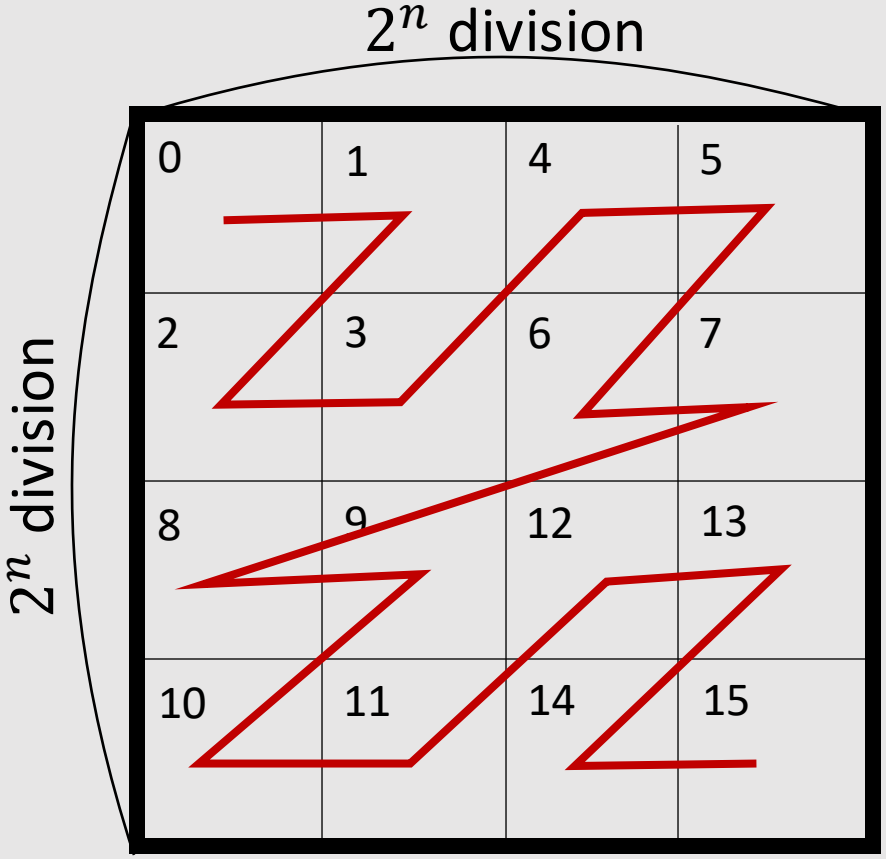
# Top-down Approach to Build BVH

- Use PCA for separating triangles into two groups



# Linear BVH: Fully Parallel Construction

- Construct BVH based on Morton code (i.e., Z-order curve)
- Two cells with close Morton codes tends to be near



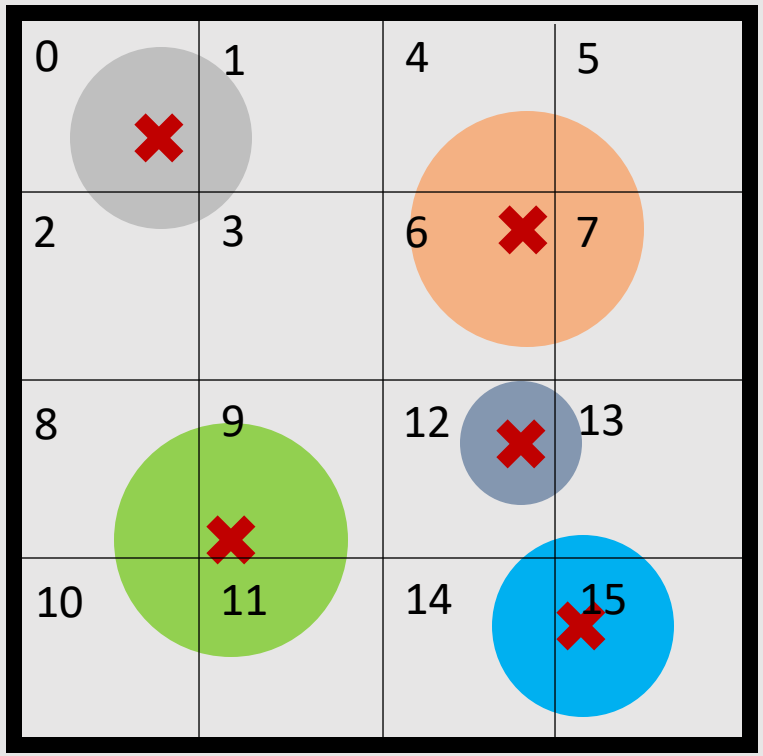

2D square domain with $2^n$ edge division

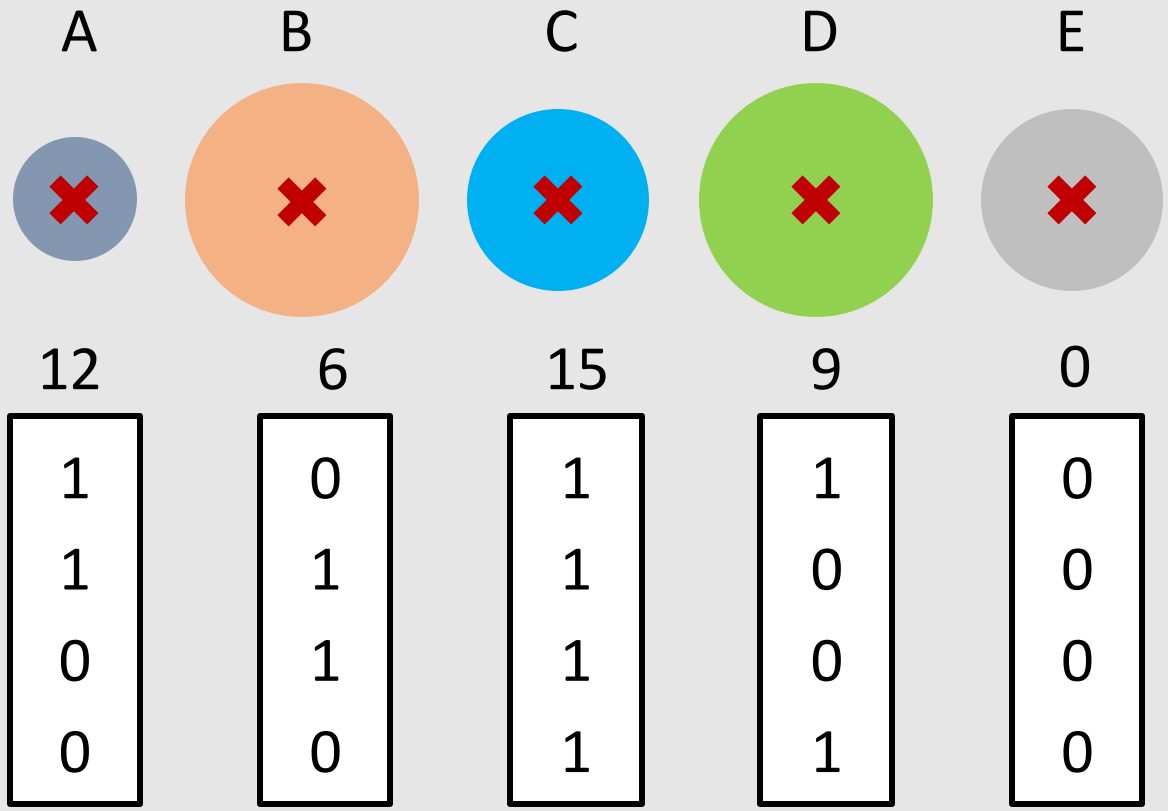
$2^{2n}$ number of cells

Cell index is size of $2n$ in binary

# Linear BVH: Fully Parallel Construction

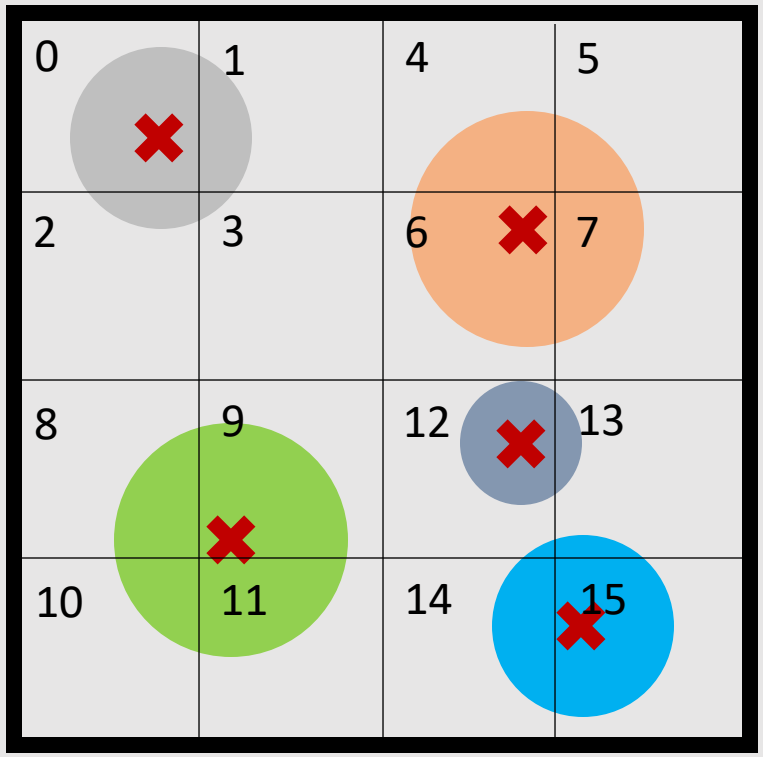
- Convert XYZ coordinate into 1D (linear) integer coordinate

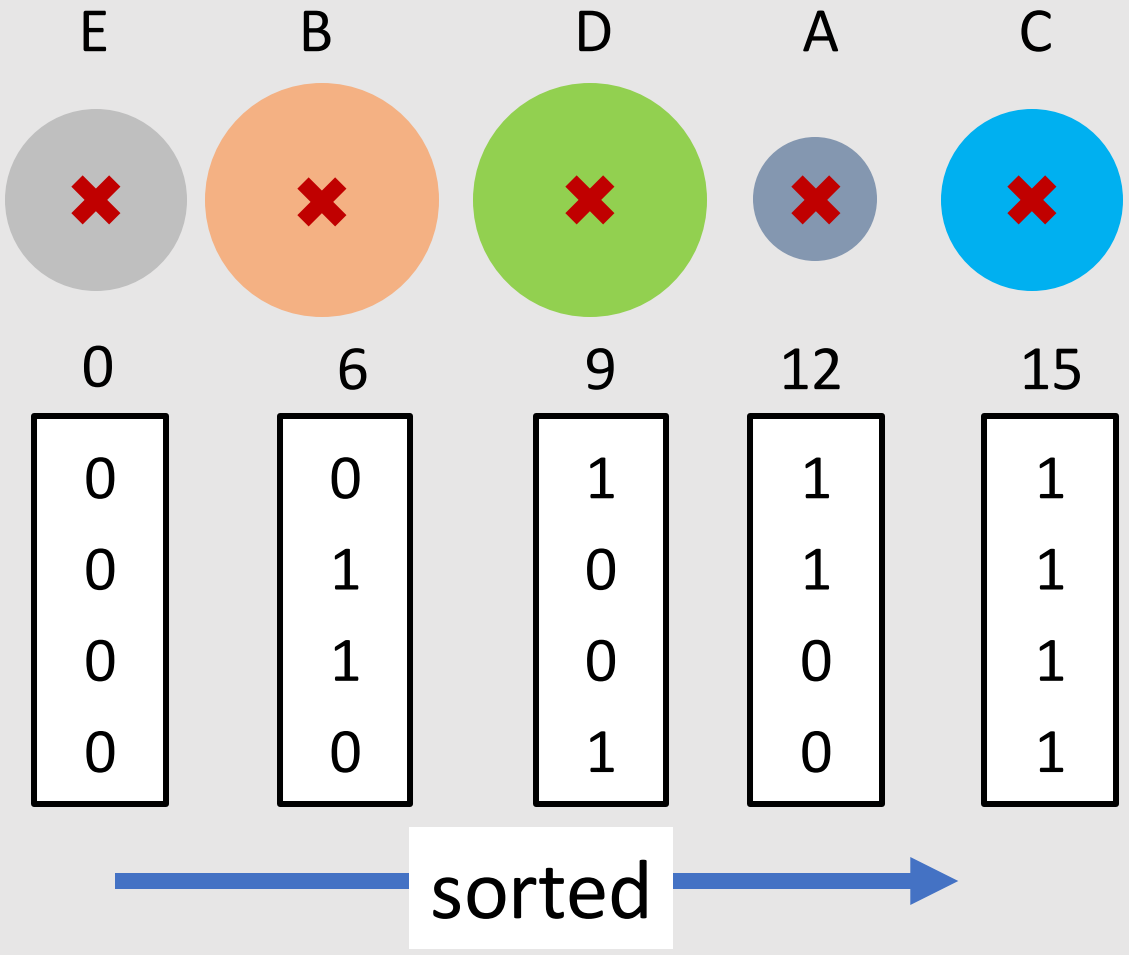


# Linear BVH: Fully Parallel Construction

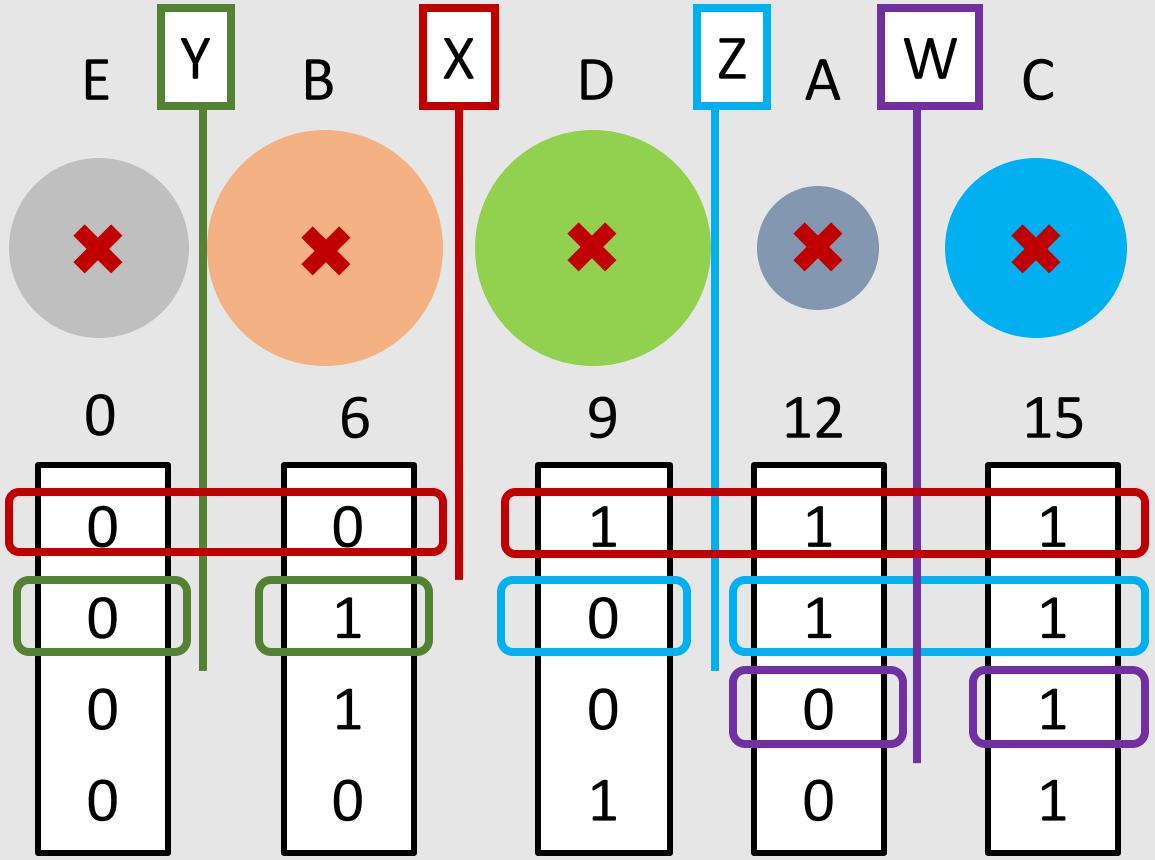
- Sort objects by their Morton codes

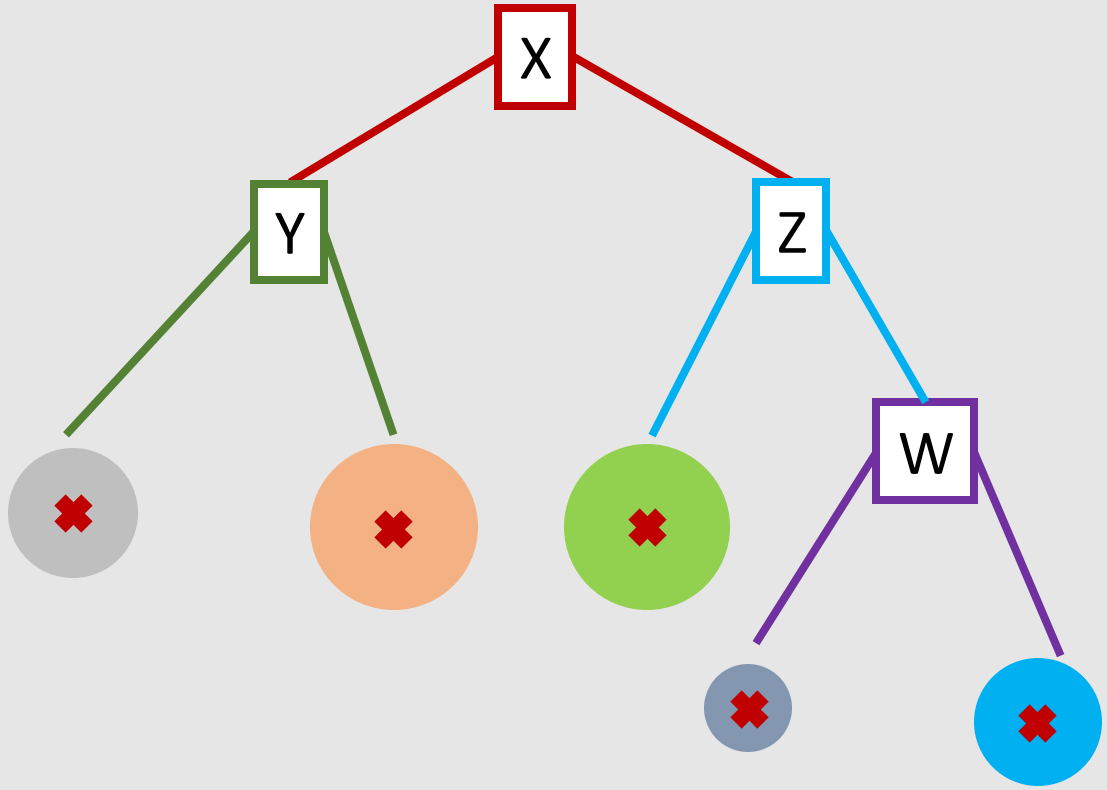


# From Morton Code to BVH Tree

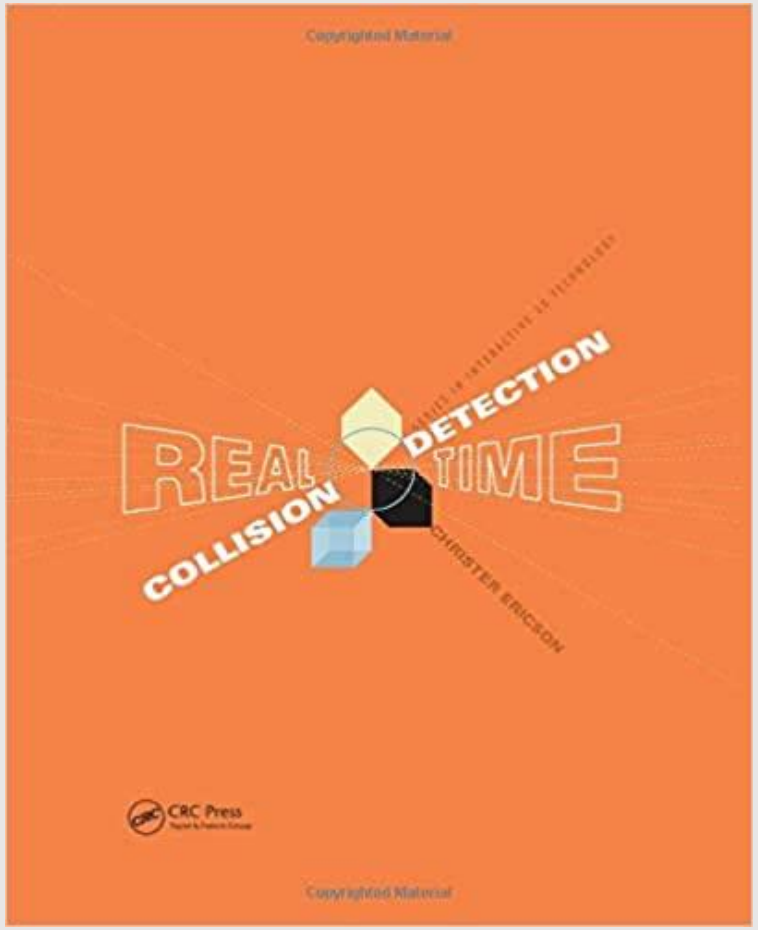
- Divide tree when digits of sorted Morton codes are different



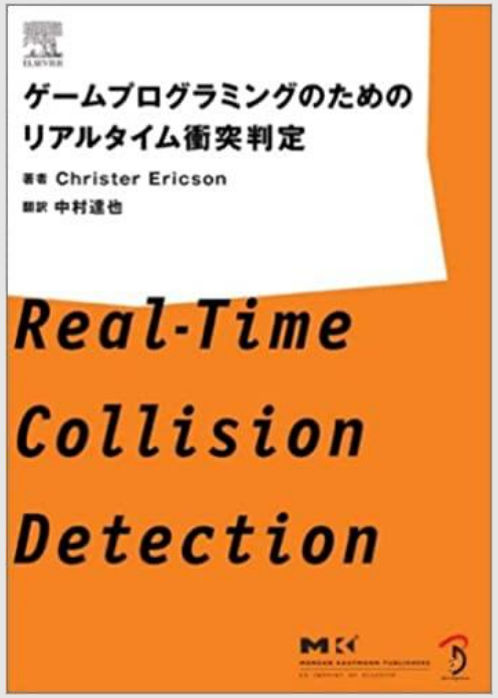



# Reference

- “Real-Time Collision Detection” by Christer Ericson




**Japanese translation available**



# Reference

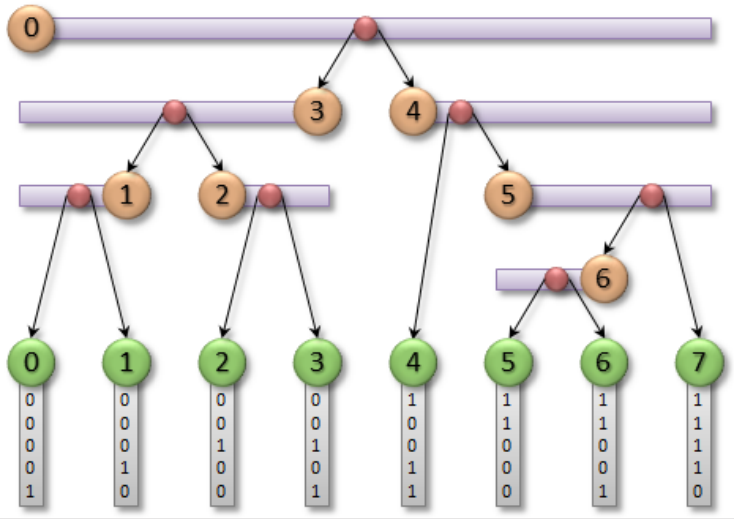
- GPU Gems 3: Chapter 32. Broad-Phase Collision Detection with CUDA


Available for free at: https://developer.nvidia.cn/gpugems/gpugems3/part-v-physics-simulation/chapter-32-broad-phase-collision-detection-cuda

# Reference on Linear-BVH

- Thinking Parallel, Part III: Tree Construction on the GPU

  by Tero Karras




https://developer.nvidia.com/blog/thinking-parallel-part-iii-tree-construction-gpu/

# Further Study

- GJK algorithm
- EPA algorithm
- Surface area heuristic