

Angular Velocity

角速度

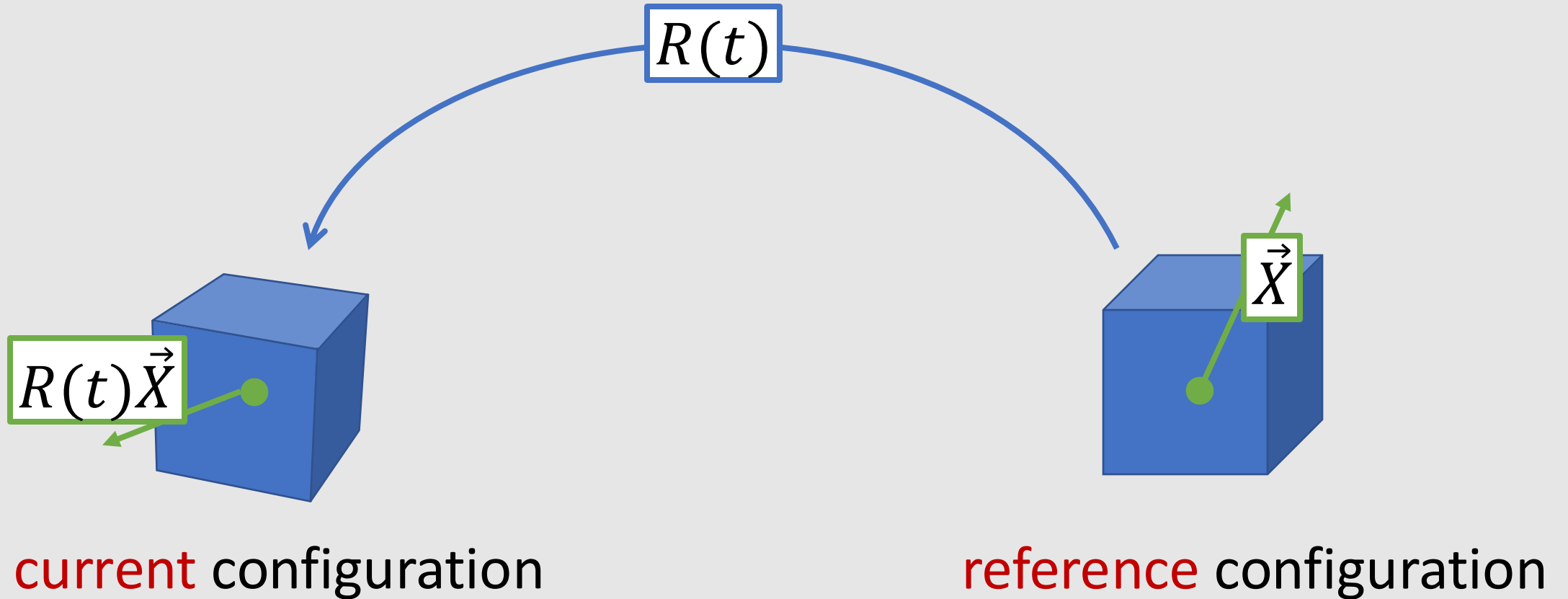
What is Skew Symmetric Matrix?

- 3x3 skew symmetric matrix represents a vector

$$\boxed{A^T = -A} \quad \Rightarrow \quad A = \text{Skew}(\vec{a}) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$\text{Skew}(\vec{a}) \vec{b} = \vec{a} \times \vec{b}$$

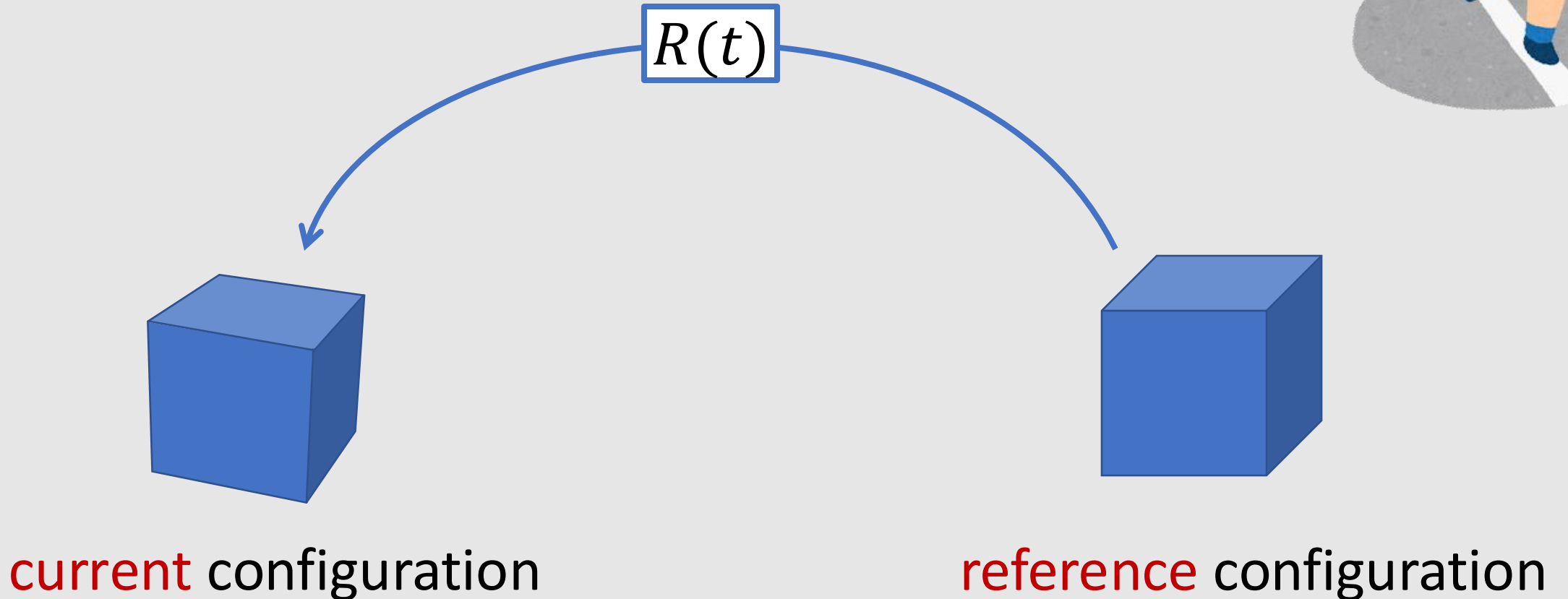
Reference & Current Configuration



walking on the edge!

Constraint on the Rotation

Rotation $R(t)$ changes under constraint $R(t)^T R(t) = I$



Differentiation of Rotation Matrix

Continuous change under constraints $R(t) \rightarrow$ DoF elimination

walking on the edge!



$R(t)$



$$R(t)^T R(t) = I$$

Differentiation of Rotation Matrix

Continuous change under constraints $R(t) \rightarrow$ DoF elimination

$$\frac{d}{dt}(RR^T) = 0$$



$$\dot{R}R^T + R\dot{R}^T = 0$$

$$\dot{R}R^T + (\dot{R}R^T)^T = 0$$

$\dot{R}R^T$ is skew-symmetric

$$\dot{R}R^T = \text{Skew}(\vec{\omega})$$

$$\dot{R} = \text{Skew}(\vec{\omega})R$$



Another Differentiation of Rotation Matrix

Continuous change under constraints $R(t) \rightarrow$ DoF elimination

$$\frac{d}{dt}(RR^T) = 0$$



$$\dot{R}R^T + R\dot{R}^T = 0$$

$$\dot{R}R^T + (\dot{R}R^T)^T = 0$$

$\dot{R}R^T$ is skew-symmetric

$$\dot{R}R^T = \text{Skew}(\vec{\omega})$$

$$\dot{R} = \text{Skew}(\vec{\omega})R$$

$$\frac{d}{dt}(R^T R) = 0$$



$$\dot{R}^T R + R\dot{R} = 0$$

$$(R\dot{R})^T + R\dot{R} = 0$$

$R\dot{R}$ is skew-symmetric

$$R^T \dot{R} = \text{Skew}(\vec{\Omega})$$

$$\dot{R} = R\text{Skew}(\vec{\Omega})$$

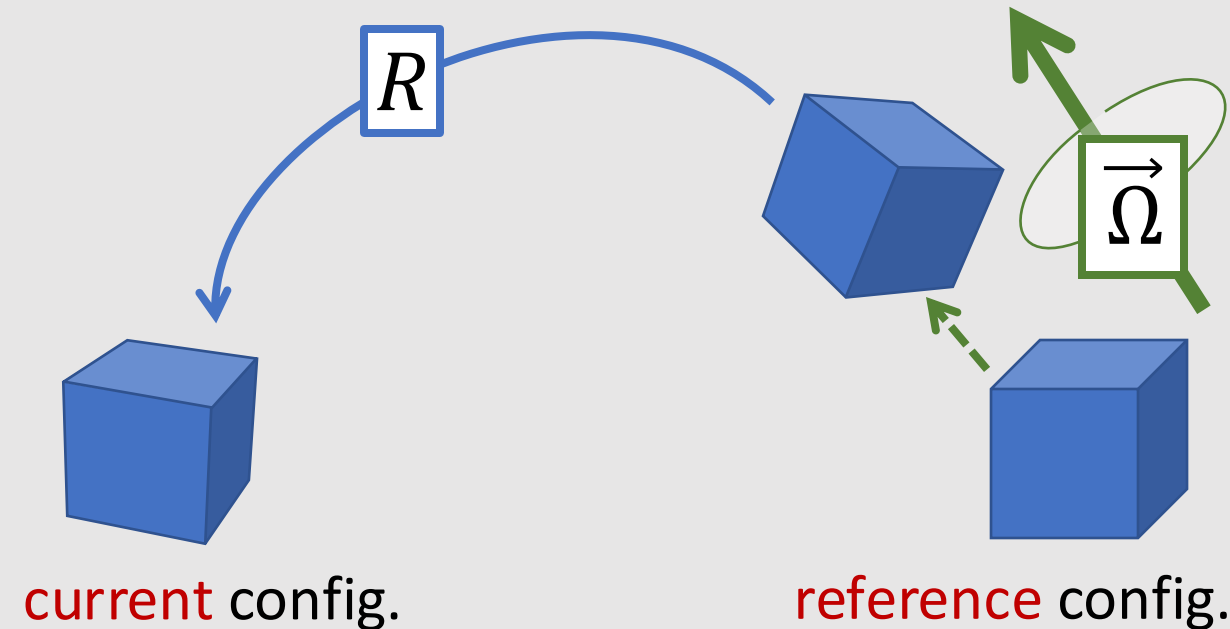
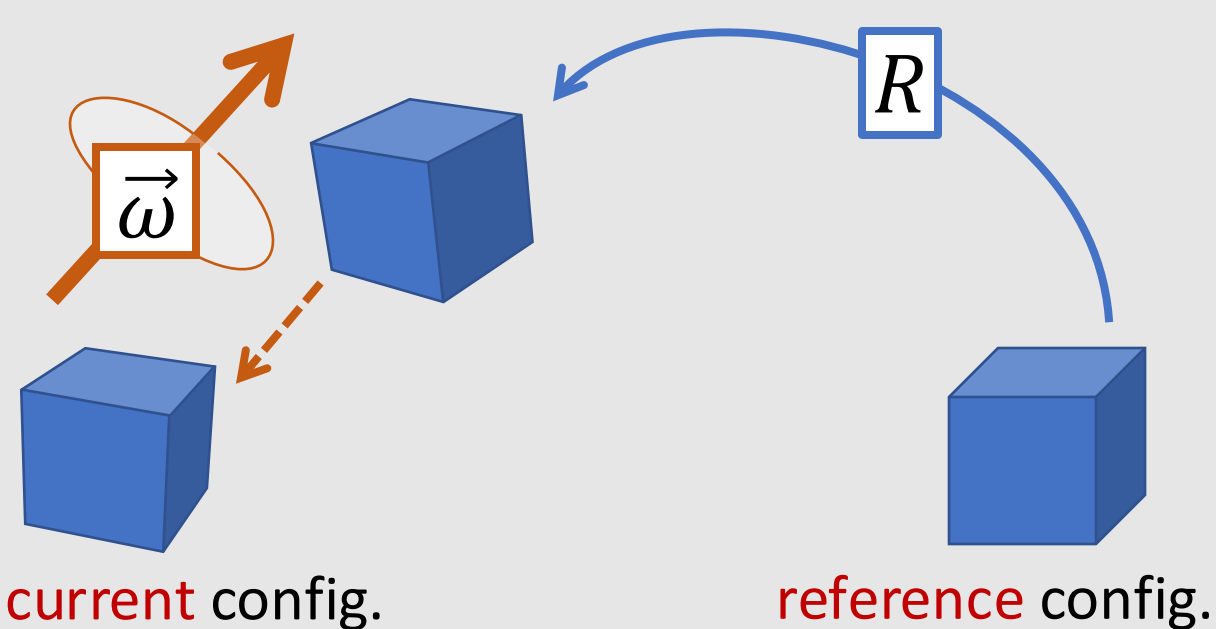
Angular Velocities are Vector Values

$\vec{\omega}$: velocity in current config.

$$\dot{R} = \text{Skew}(\vec{\omega})R$$

$\vec{\Omega}$: velocity in reference config.

$$\dot{R} = R \text{Skew}(\vec{\Omega})$$



Relationship between Two Angular Velocities

$\vec{\omega}$: velocity in current config.

$$\dot{R} = \text{Skew}(\vec{\omega})R$$

$\vec{\Omega}$: velocity in reference config.

$$\dot{R} = R \text{Skew}(\vec{\Omega})$$

$$\text{Skew}(\vec{\omega}) = R \text{Skew}(\vec{\Omega})R^T$$

$$\vec{\omega} = R \vec{\Omega}$$



Angular Velocities are Vector Values

$\vec{\omega}$: velocity in current config.

$$\dot{R} = \text{Skew}(\vec{\omega})R$$

$\vec{\Omega}$: velocity in reference config.

$$\dot{R} = R \text{Skew}(\vec{\Omega})$$

