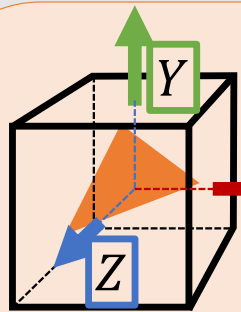
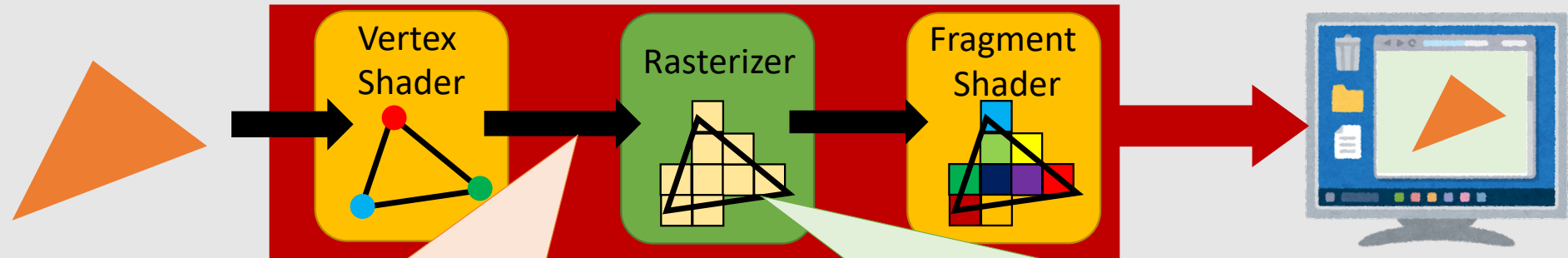


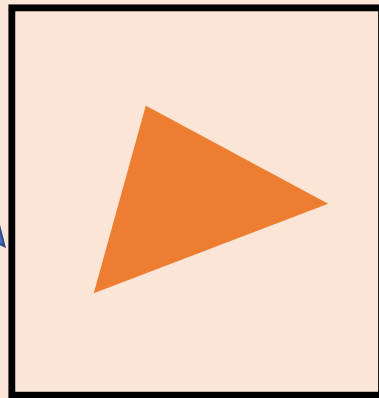
# **Rasterization**

# Rasterization

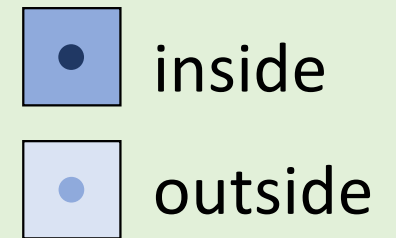
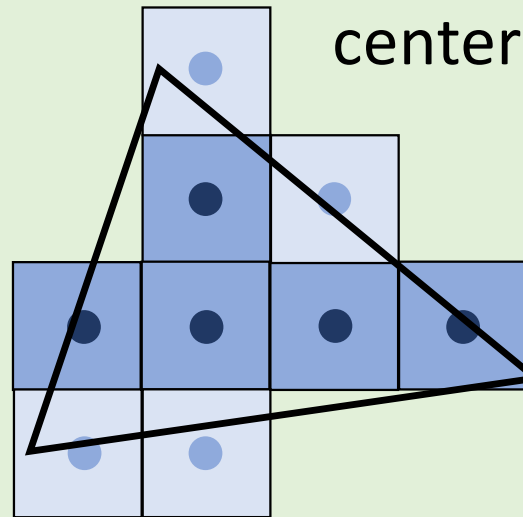


canonical  
view volume

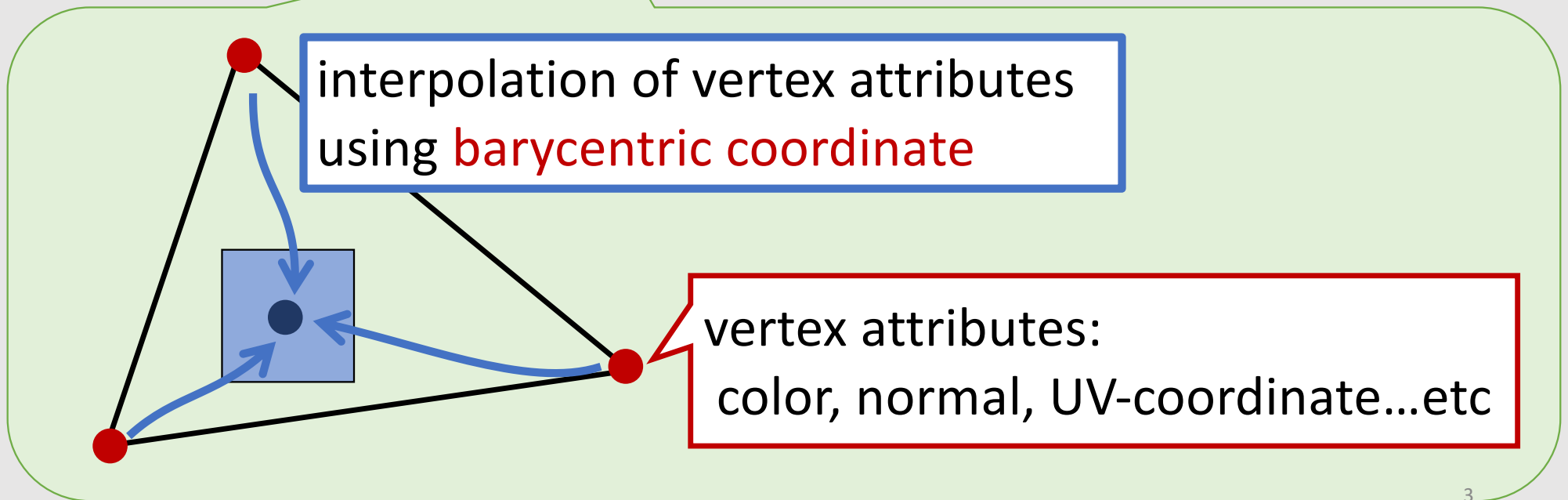
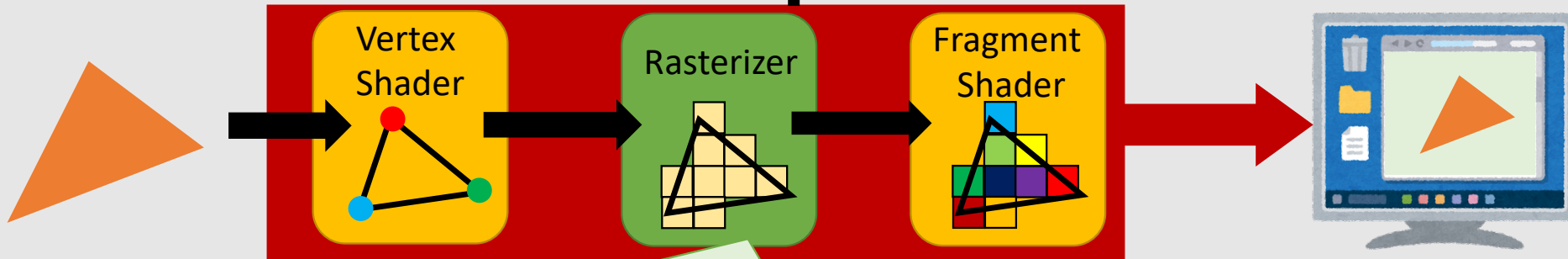
Looking  
from  $Z^{+\infty}$



Extract the pixels whose  
center is inside the triangle



# Rasterizer and Interpolation



# Why You Need to Understand Rasterization?

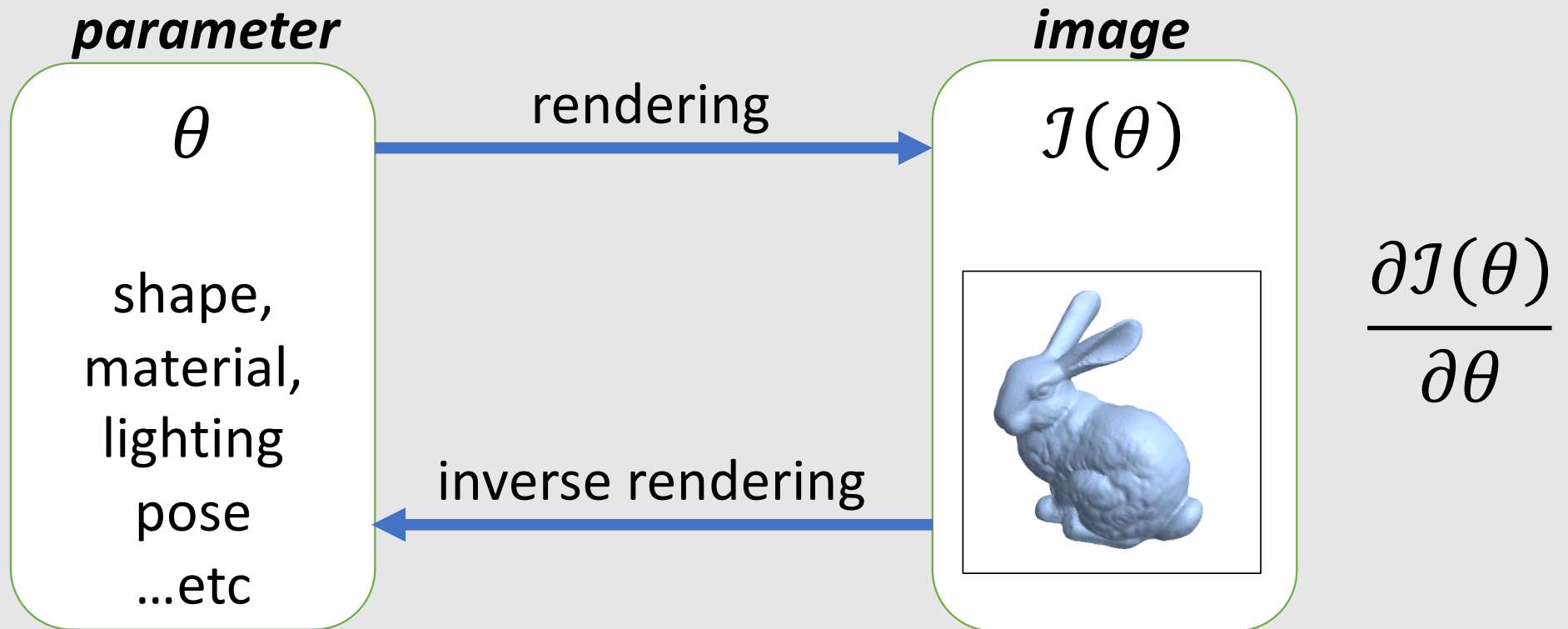
- Understand modern gaming engine architecture



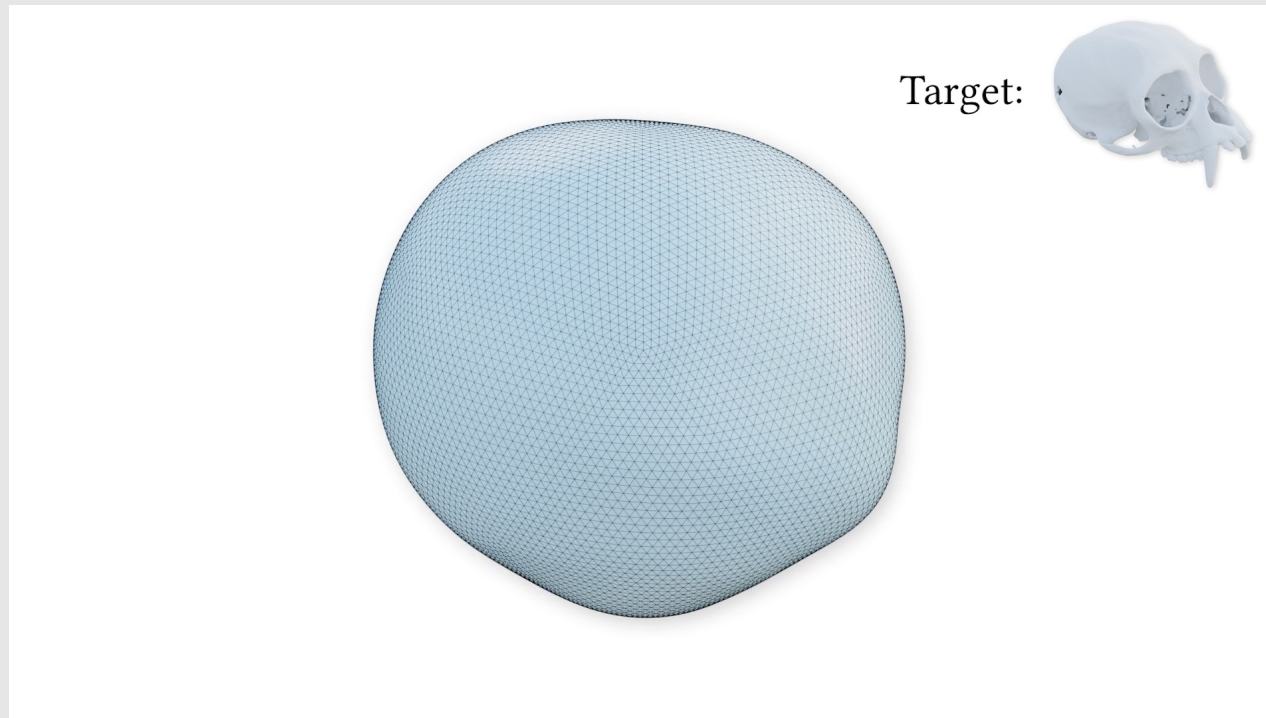
© 2004-2020, Epic Games, Inc. All rights reserved. Unreal and its logo are Epic's trademarks or registered trademarks in the US and elsewhere.

# Why You Need to Understand Rasterization?

- Differentiable rendering, inverse rendering



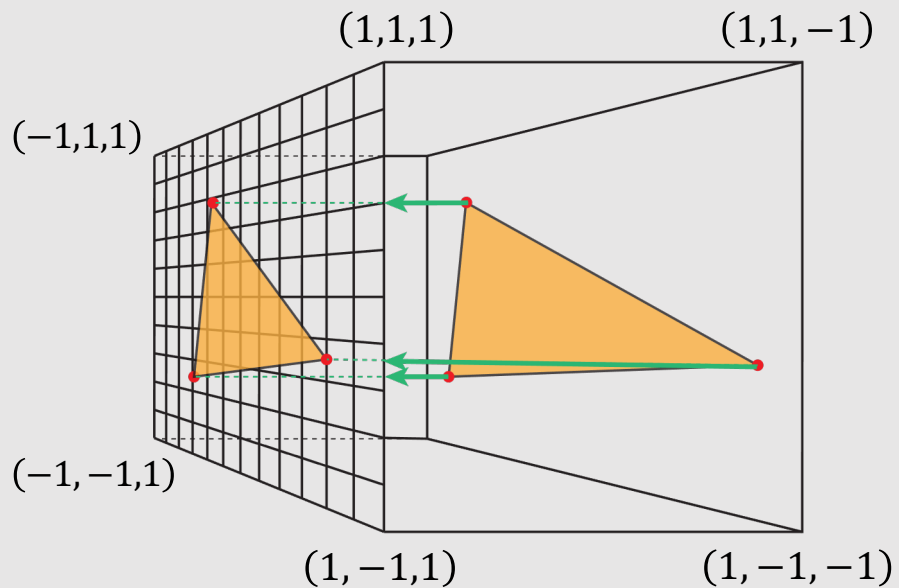
# Example of Differentiable Rendering



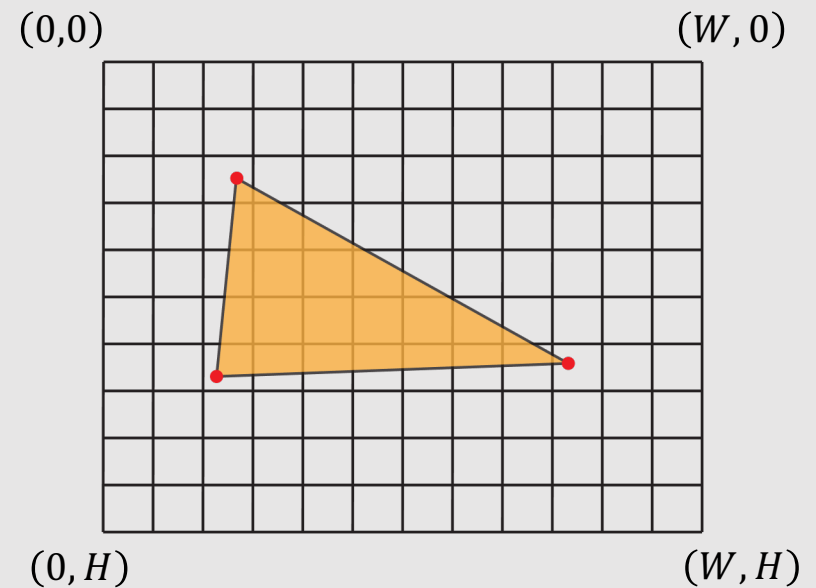
Large Steps in Inverse Rendering of Geometry,  
Baptiste Nicolet Alec Jacobson Wenzel Jakob  
In ACM Transactions on Graphics (Proceedings of SIGGRAPH Asia 2021)

# Rasterization: The First Step

Orthogonal projection in canonical view volume

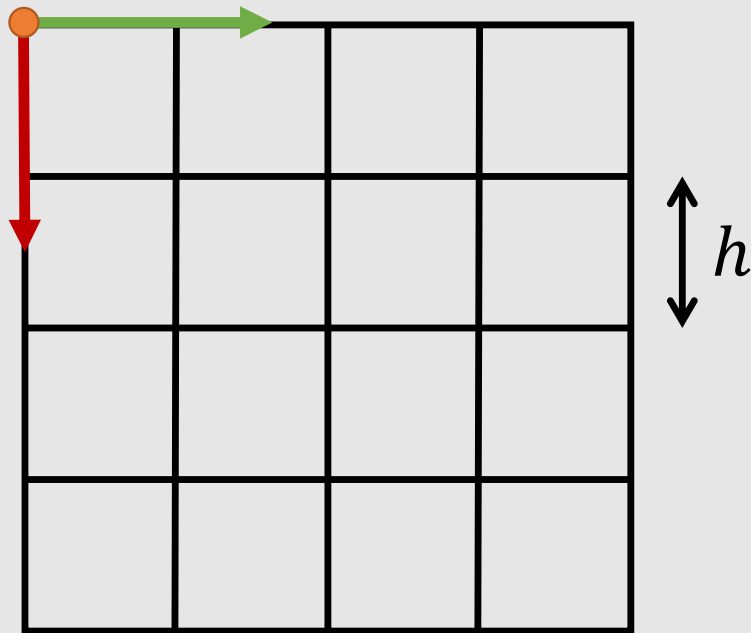


Triangle in the image coordinate



# Regular Grids

- Most common discretization for spatial values



Let's find out the corresponding grid cell for  $(p_x, p_y)$

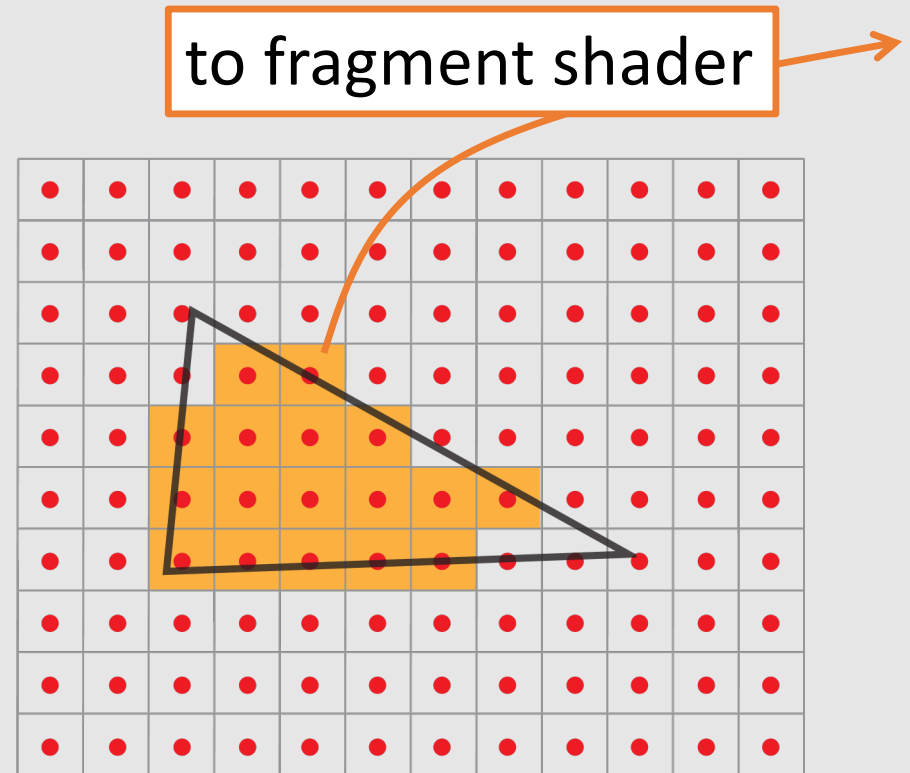
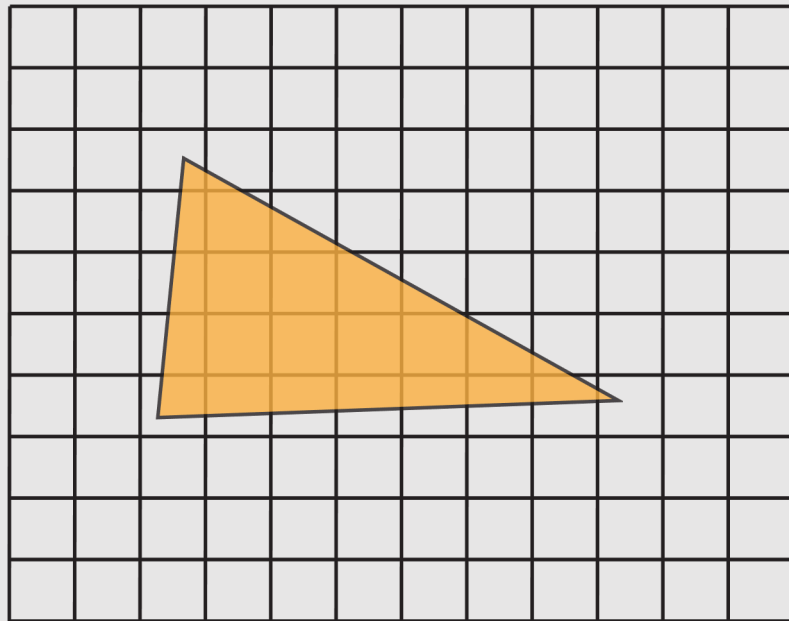
Check it out!





# Inside & Outside Test at the Center of Pixel

- Extract the pixels whose center is inside the triangle



# Triangle Inside & Outside Test

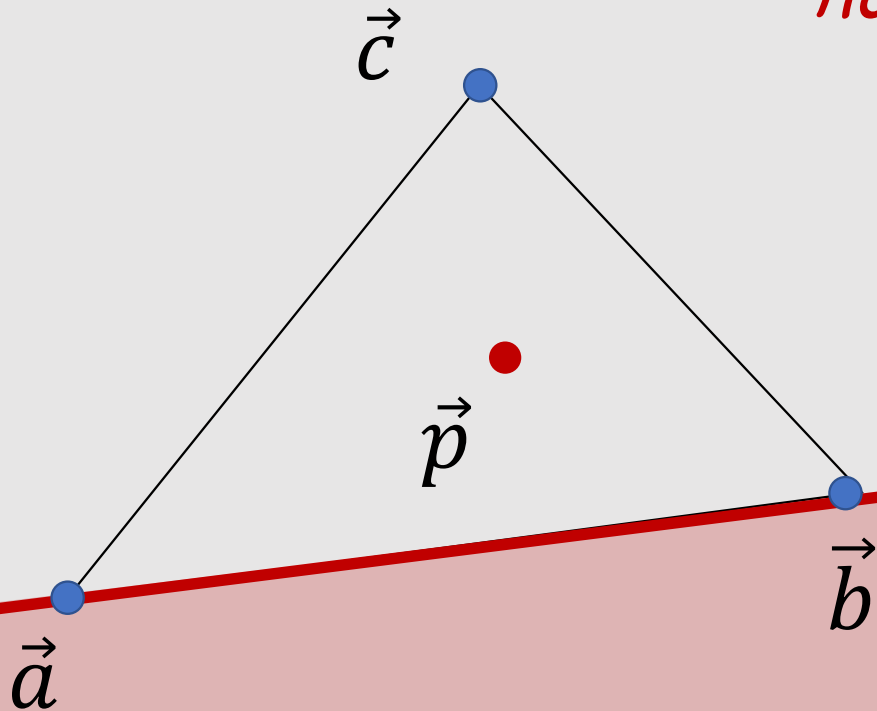
- Edge connecting  $\vec{a} = \{a_x, a_y, 1\}^T$  and  $\vec{b} = \{b_x, b_y, 1\}^T$

*homogenous coordinate!*

Left side:  $(\vec{a} \times \vec{b}) \cdot \vec{p} > 0$

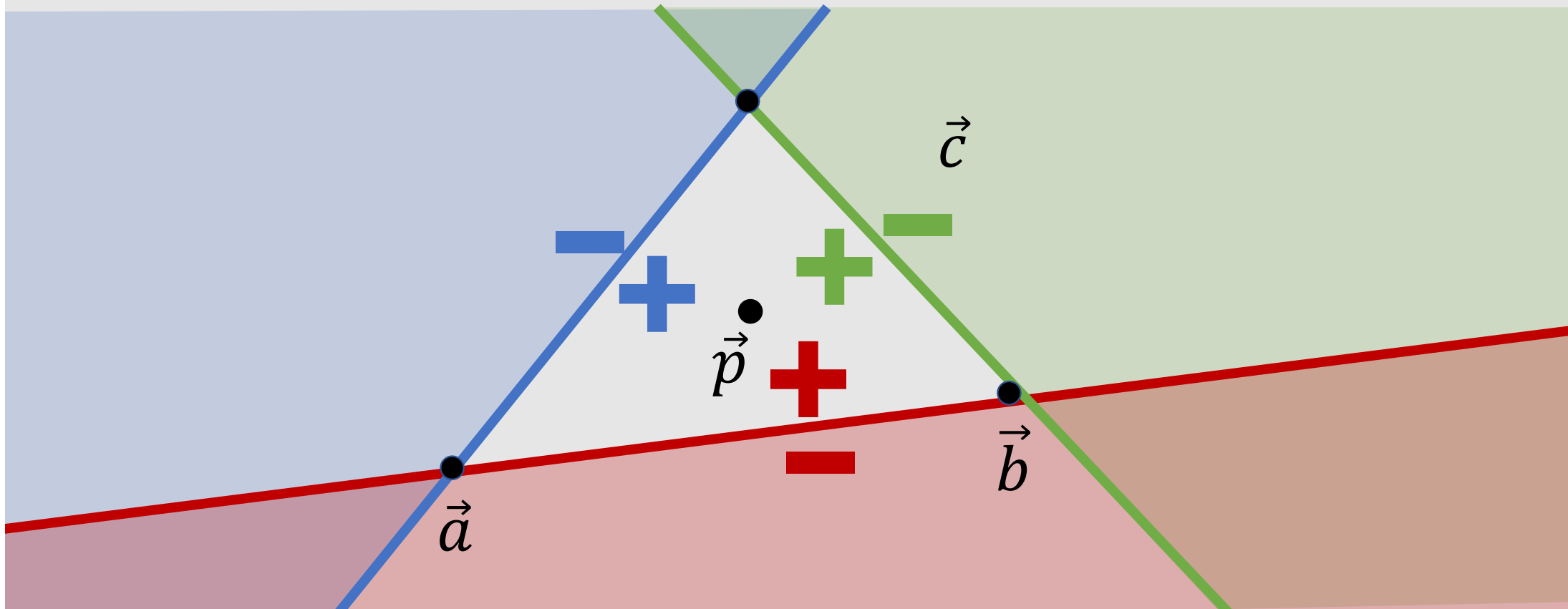
$$(\vec{a} \times \vec{b}) \cdot \vec{p} = 0$$

Right side:  $(\vec{a} \times \vec{b}) \cdot \vec{p} < 0$

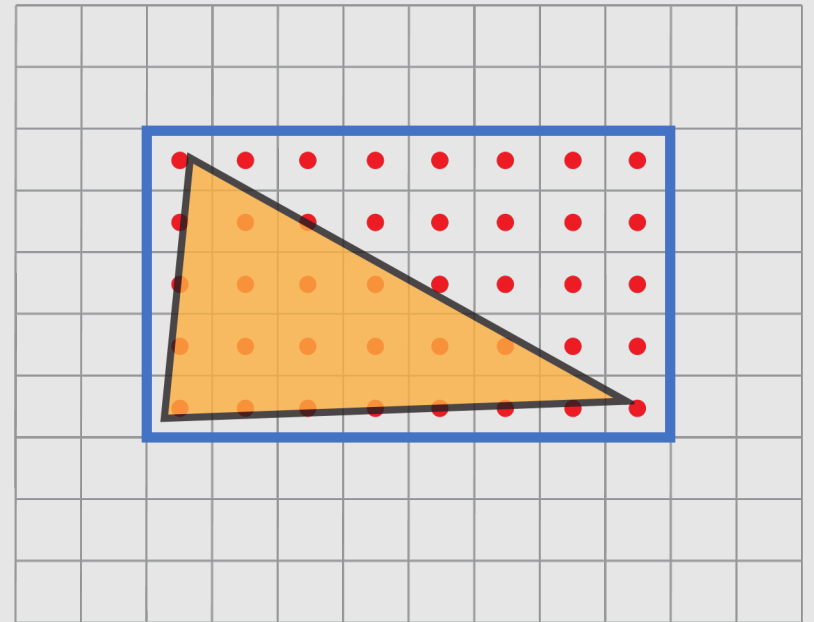
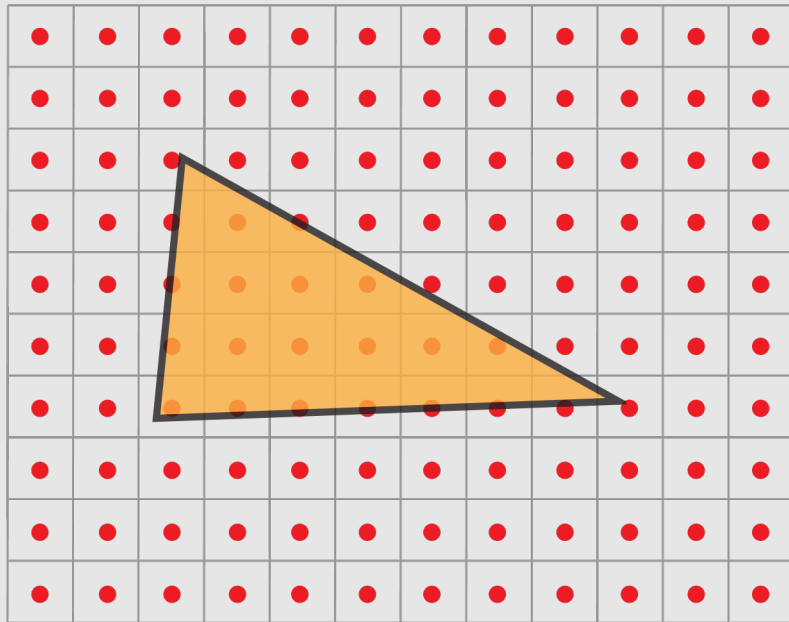


# Triangle Inside & Outside Test

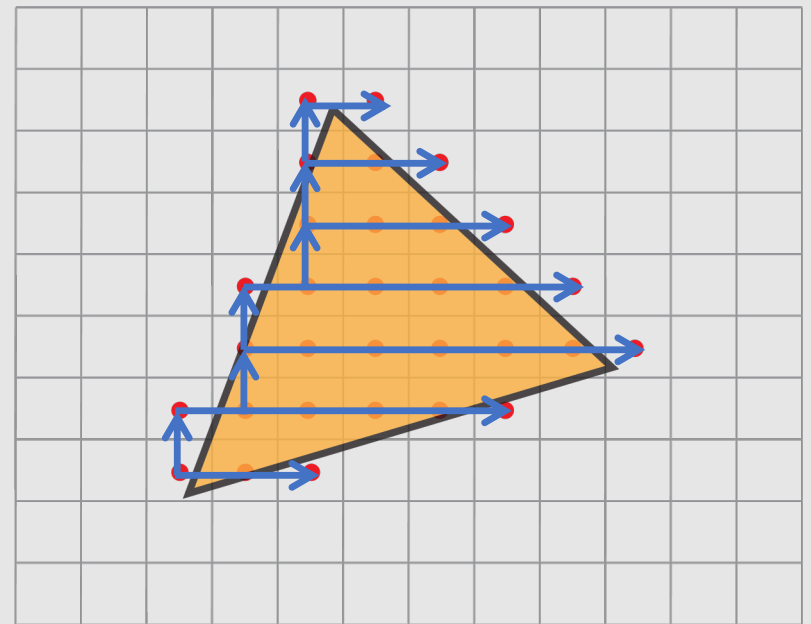
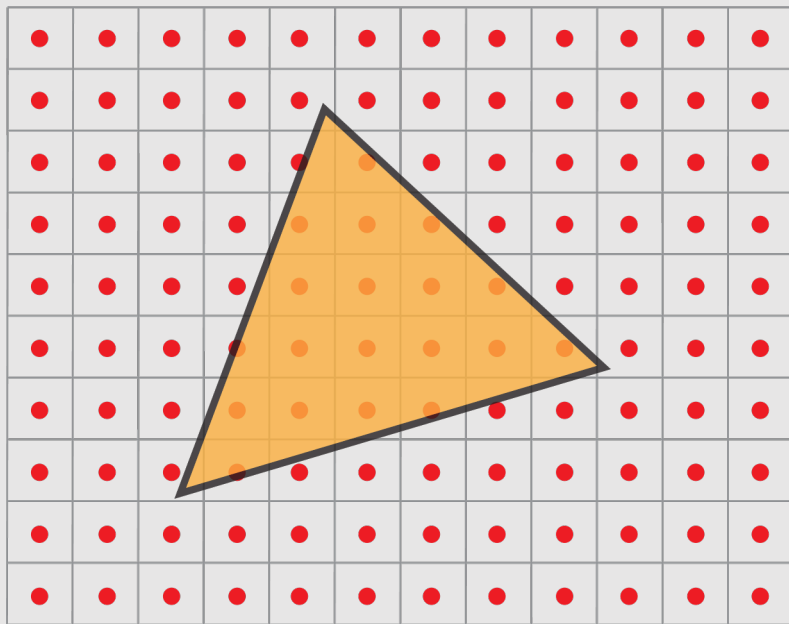
- Check if the signs are the same for all the three edges



# Optimizing Test 1: Bounding Box



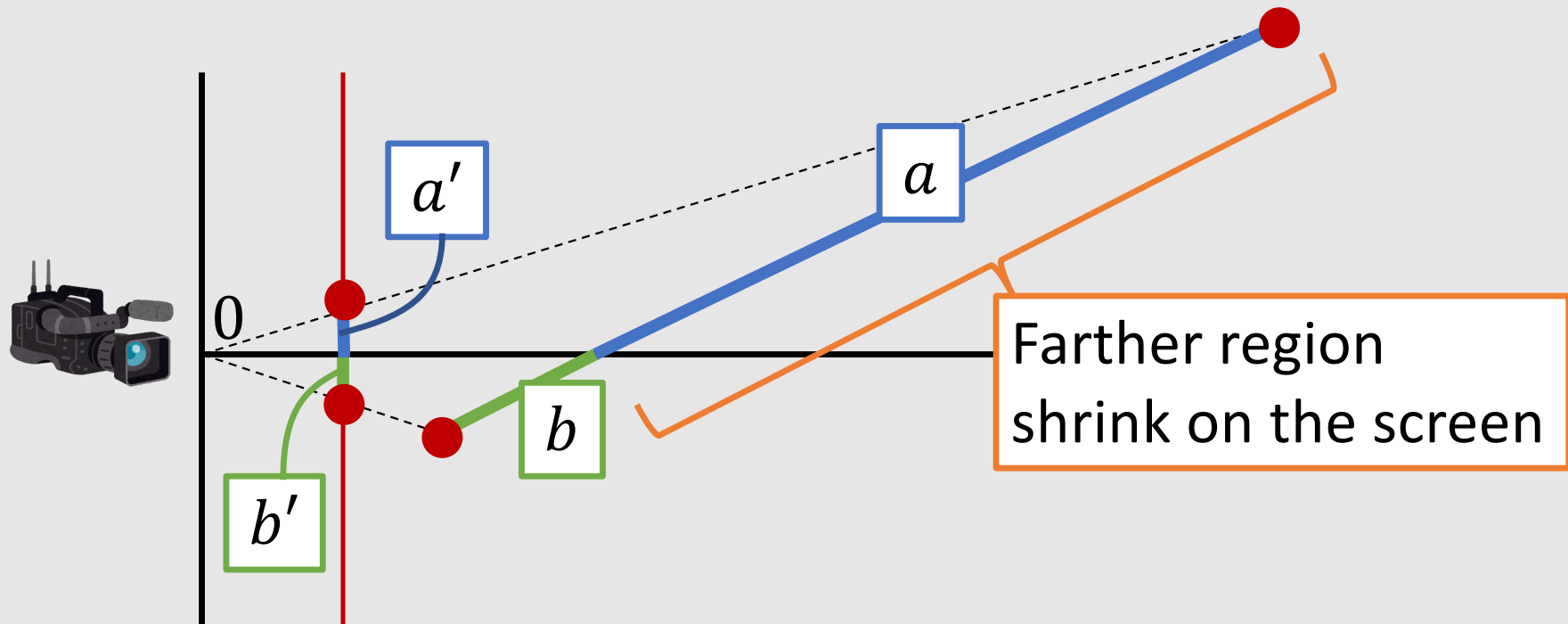
# Optimizing Test 2: Pineda's Algorithm



Juan Pineda. 1988. A parallel algorithm for polygon rasterization. In Proceedings of the 15th annual conference on Computer graphics and interactive techniques (SIGGRAPH '88).

# Distortion in Perspective Interpolation

$$a:b \neq a':b'$$



# Interpolation on Screen vs Object

project then  
interpolate  
*"Gouraud"*  
*interpolation*

interpolate then  
project  
*perspectively correct*  
*interpolation*

top view

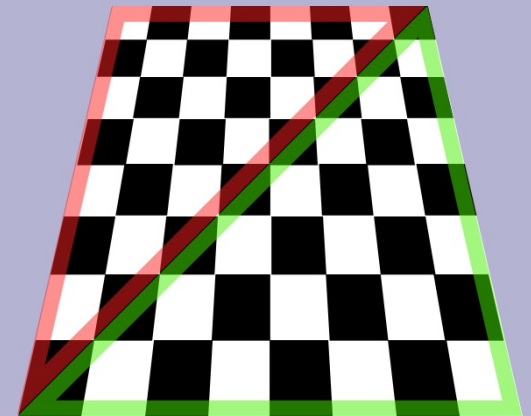
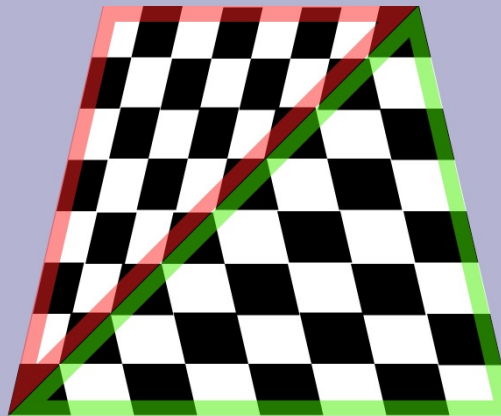
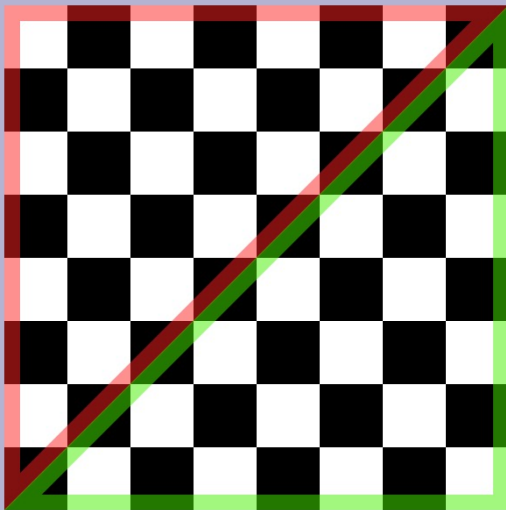
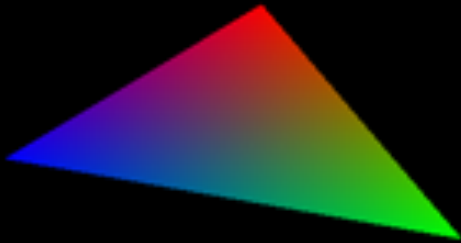


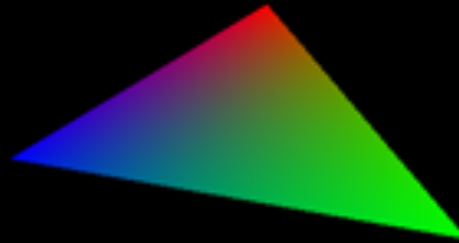
Image Credit: Darkness3560 @ Wikipedia

# Perspectively Correct Interpolation

naive interpolation (bad)



perspective correct interpolation

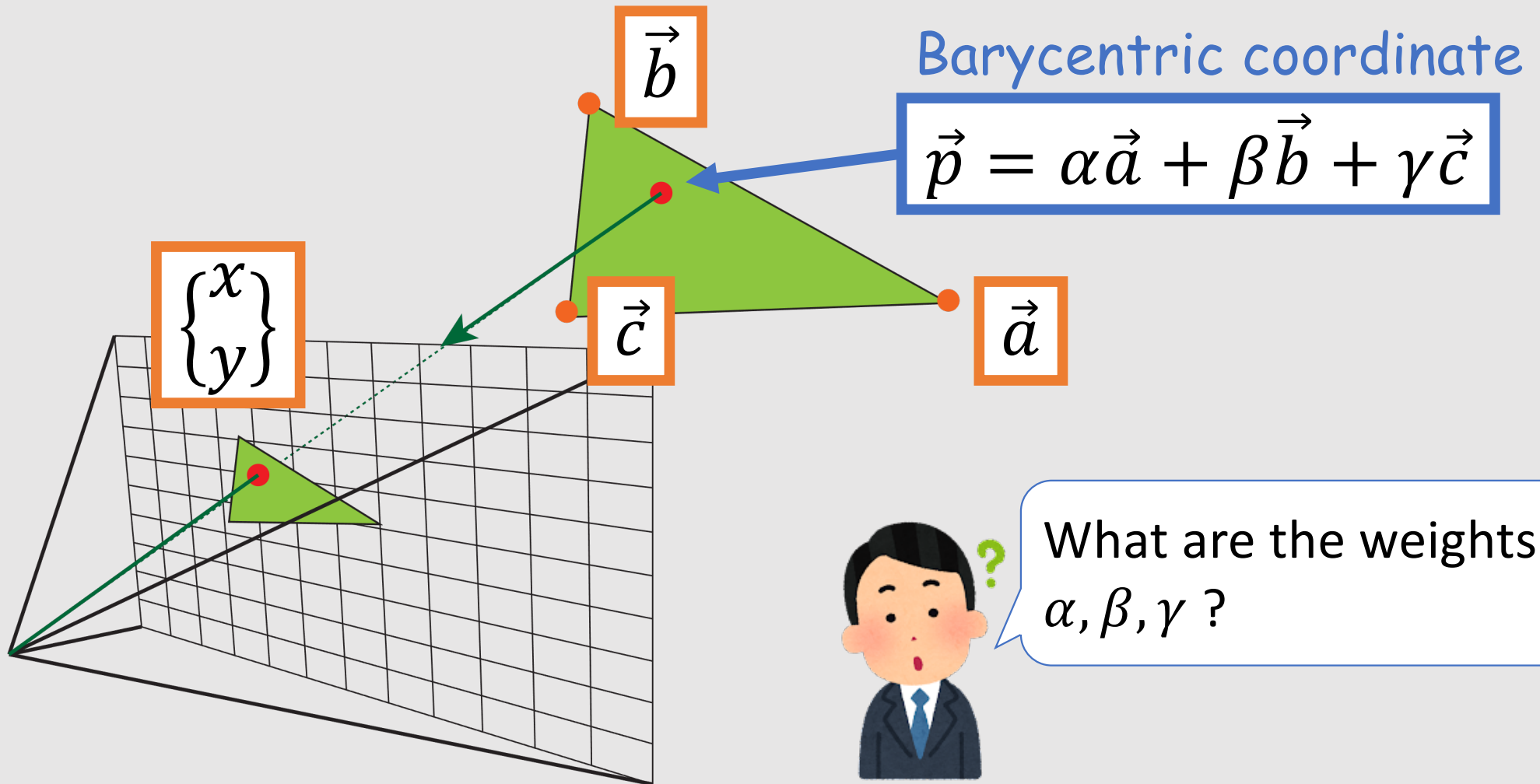


z-buffer



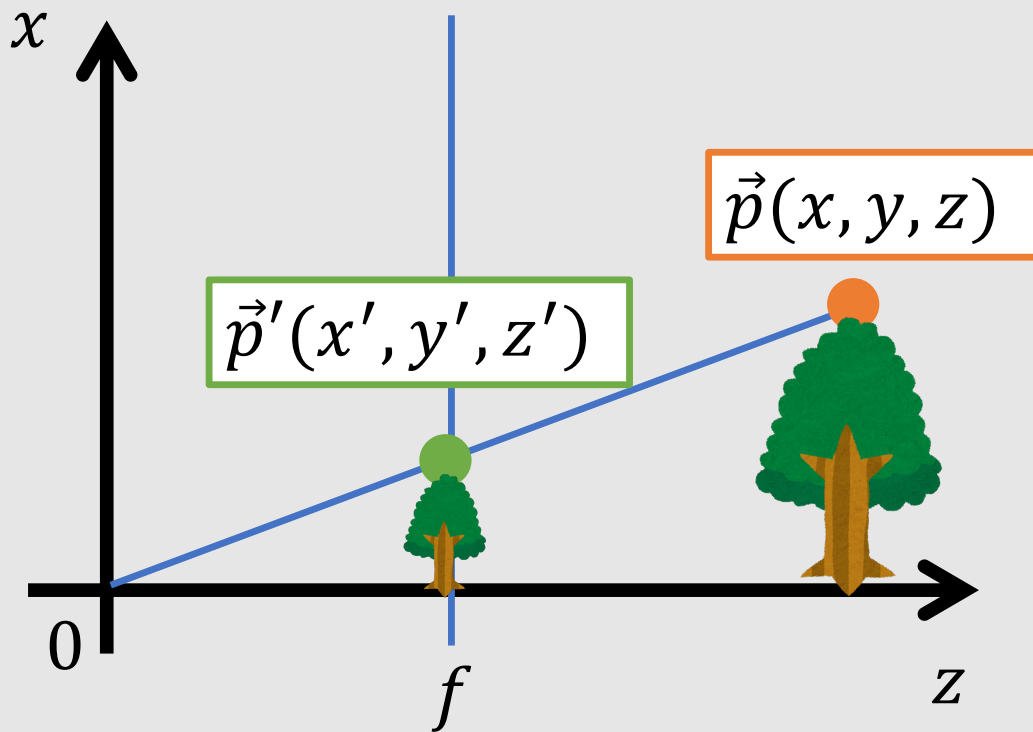


# Perspectively Correct Interpolation



# Simple Perspective

- Projecting  $\vec{p}(x, y, z)$  on the image plane  $z = f$  ( $f$ : focal length)



$$z' = f, \quad \frac{x'}{x} = \frac{y'}{y} = \frac{z'}{z}$$

$$x' = \frac{fx}{z}, \quad y' = \frac{fy}{z}$$

$$\begin{Bmatrix} x' \\ y' \\ 1 \end{Bmatrix} \propto \begin{Bmatrix} x'' \\ y'' \\ w \end{Bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

# Baricentric Weights for Interpolation

- Distributive property of matrix

$$\begin{Bmatrix} x \\ y \\ 1 \end{Bmatrix} \propto \begin{Bmatrix} x' \\ y' \\ w \end{Bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \vec{p}$$

$$= H(\alpha \vec{a} + \beta \vec{b} + \gamma \vec{c})$$

$$= (H\vec{a})\alpha + (H\vec{b})\beta + (H\vec{c})\gamma$$

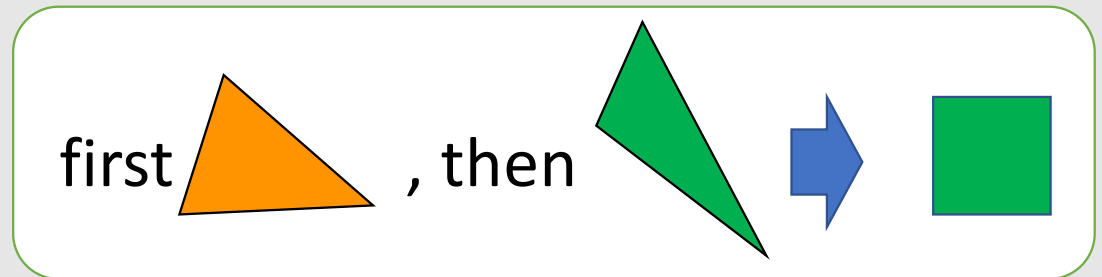
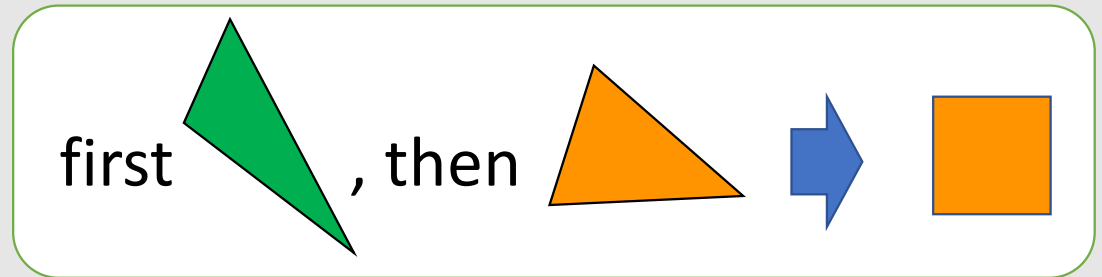
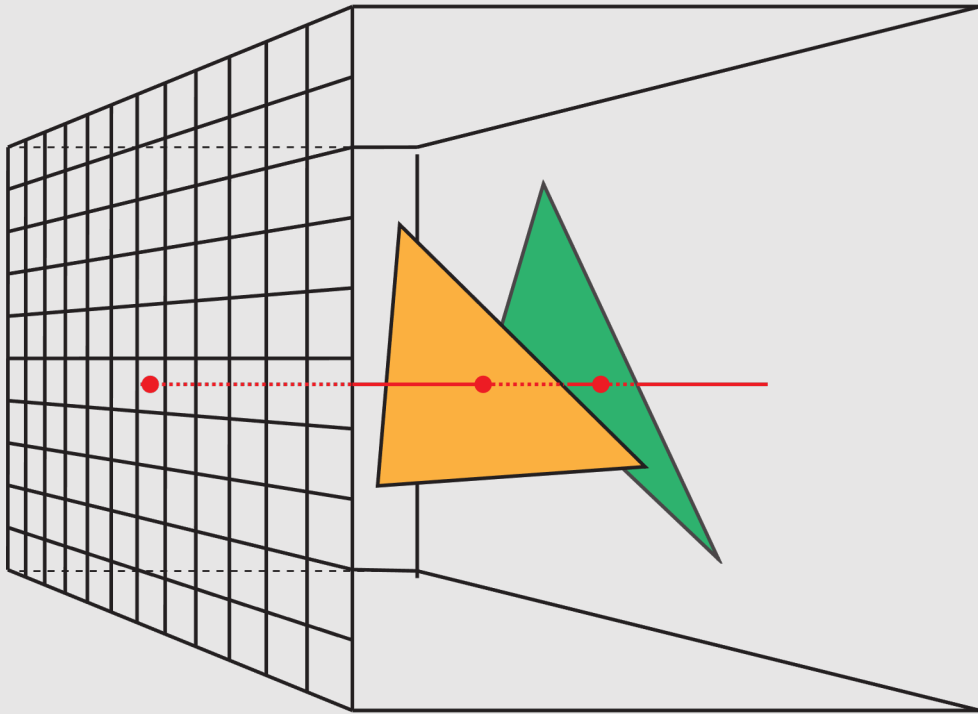
$\alpha, \beta, \gamma$  must satisfy  
 $\alpha + \beta + \gamma = 1$

4 degrees of freedom,  
4 constraints



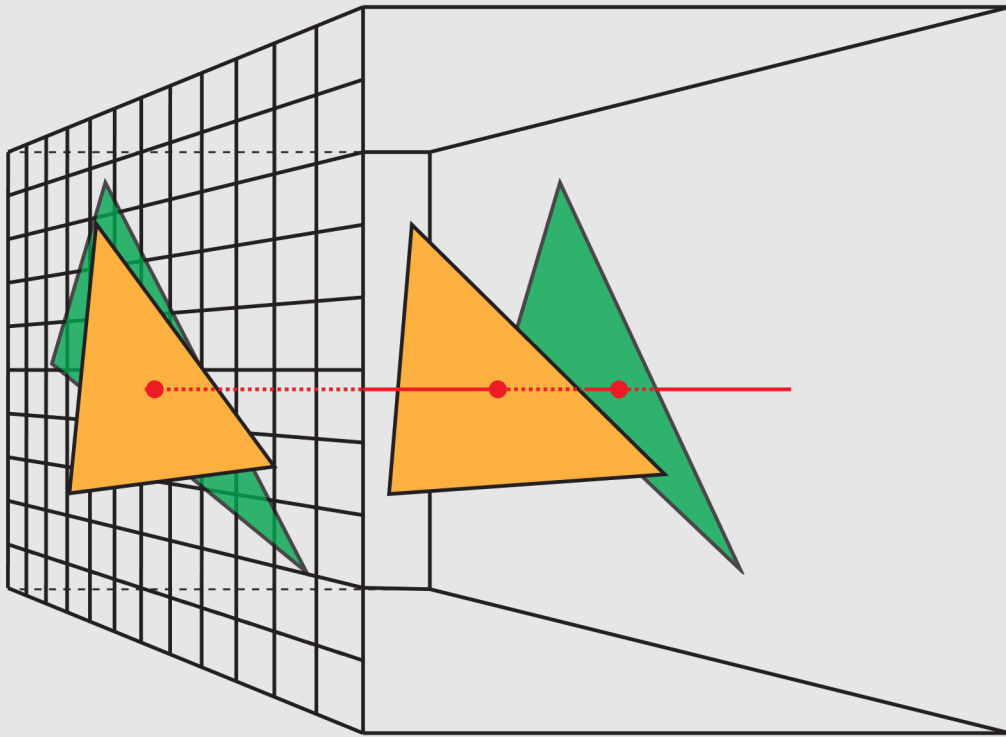
# How To Remove Order Dependency?

- Since shader is executed in *parallel*, difficult to handle *occlusion*



# Z-Buffer Algorithm

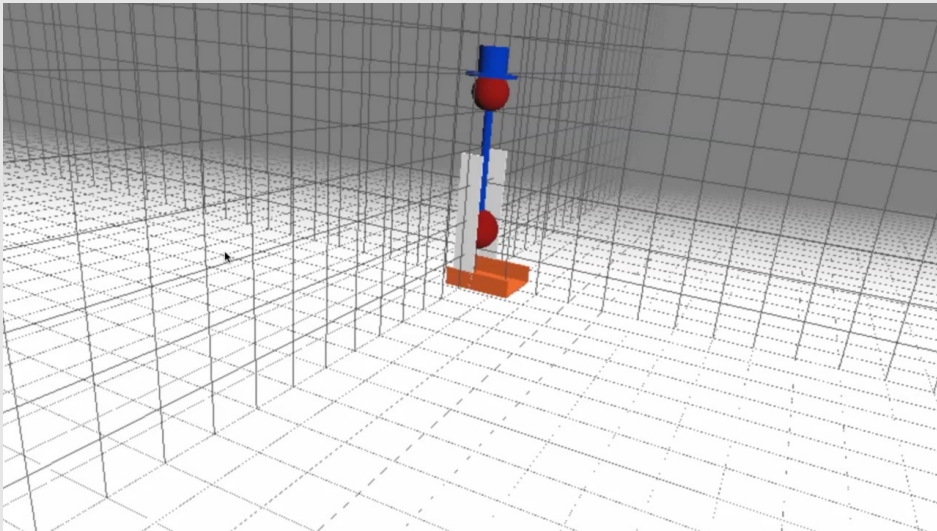
- Frame-buffer that keeps minimum depth for each pixel



```
for each triangle
  for each pixel (x,y)
    if (x,y) is inside triangle
      compute z
      if z < zbuffer[x,y]
        zbuffer[x,y]=z
        framebuffer[x,y]=shade()
```

# Z-Fighting

- Flickering when different triangles has the same (or similar) depth



Z-Fighting - Interactive 3D Graphics  
<https://www.youtube.com/watch?v=CjckWVwd2ek>

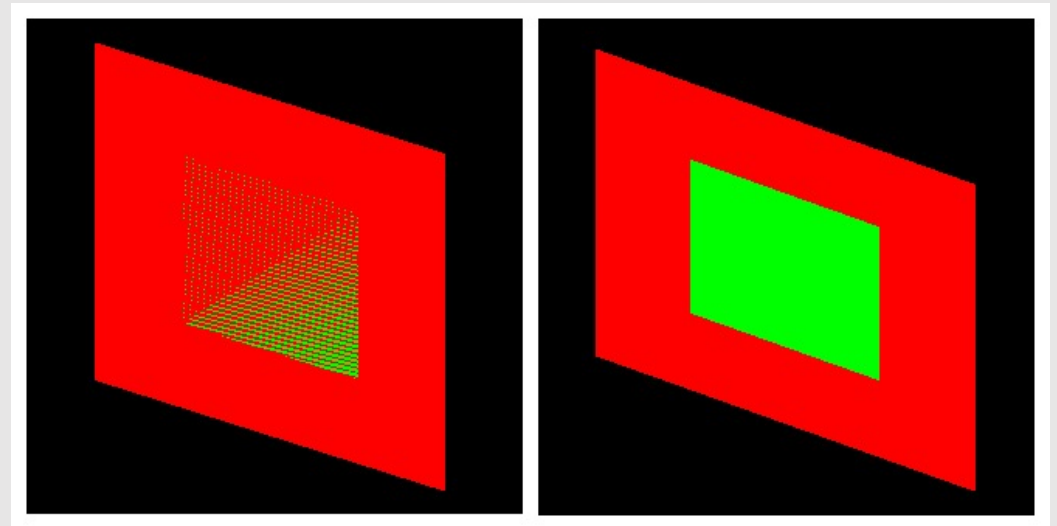
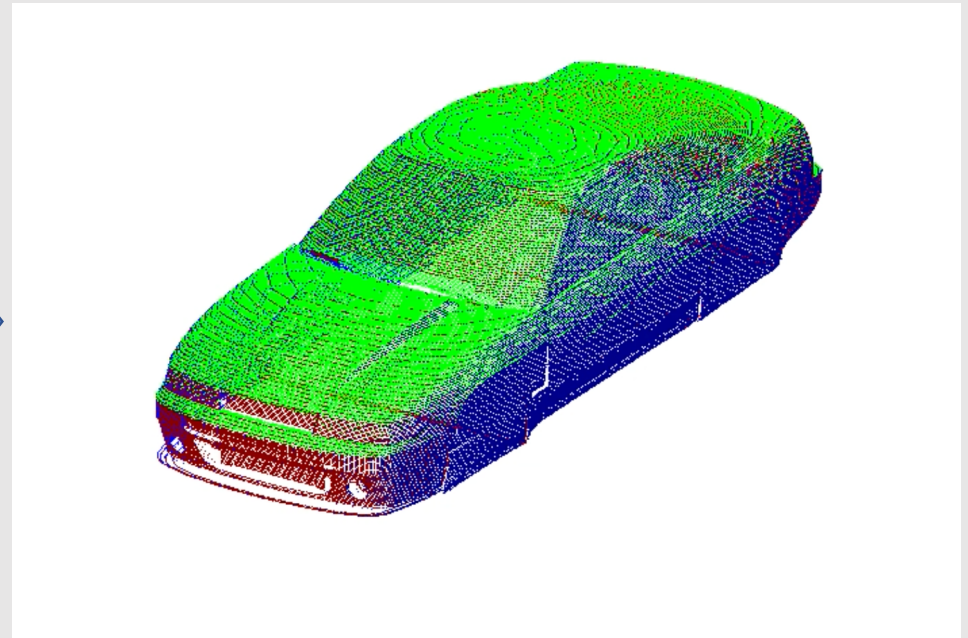


Image Credit: Wojciech Muła @ Wikipedia

# Biproduct of Z-buffer Method: Depth Image

- Depth Image can be used for various geometry processing

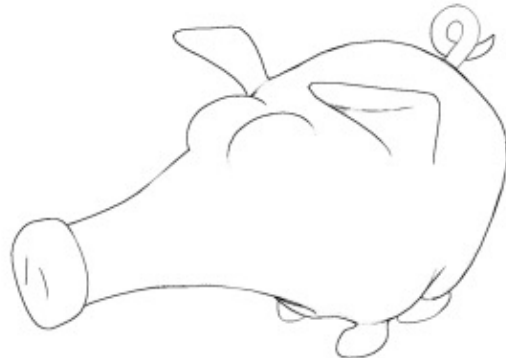


# Depth Image Usage 1: Contour Drawing

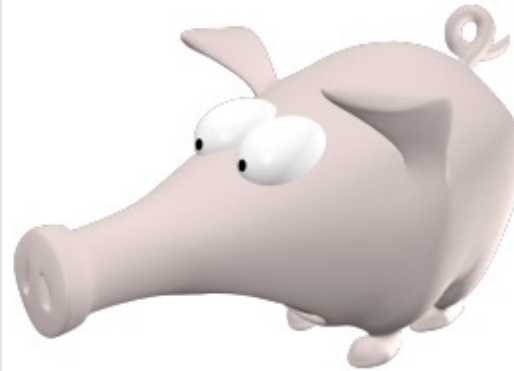
- Non-photo realistic (NPR) rendering



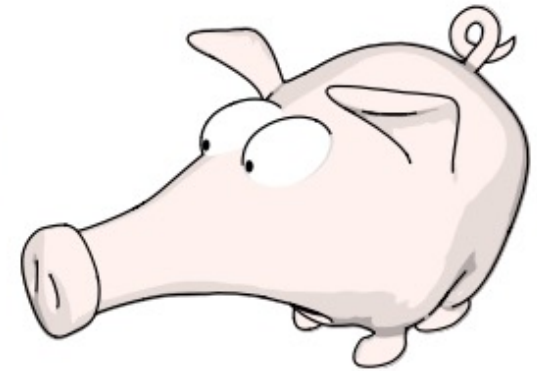
depth image



depth  
discontinuity



naïve  
diffuse shading



NPR rendering  
with contour

[1] Bénard, Pierre, and Aaron Hertzmann. "Line drawings from 3D models: A tutorial." Foundations and Trends® in Computer Graphics and Vision 11, no. 1-2 (2019): 1-159.



# Depth Image Usage 2: DoF Effect

- Depth of field (DoF, 被写界深度)

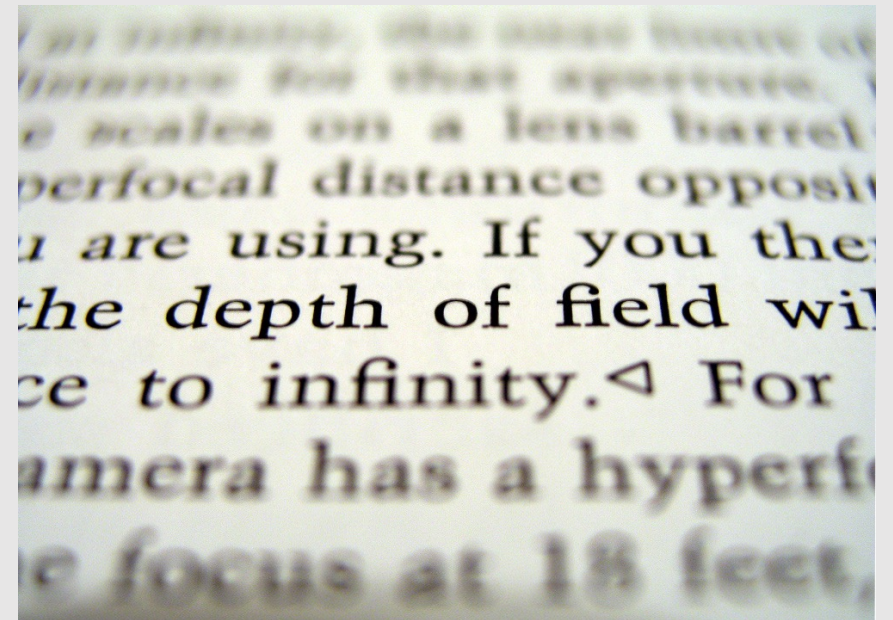
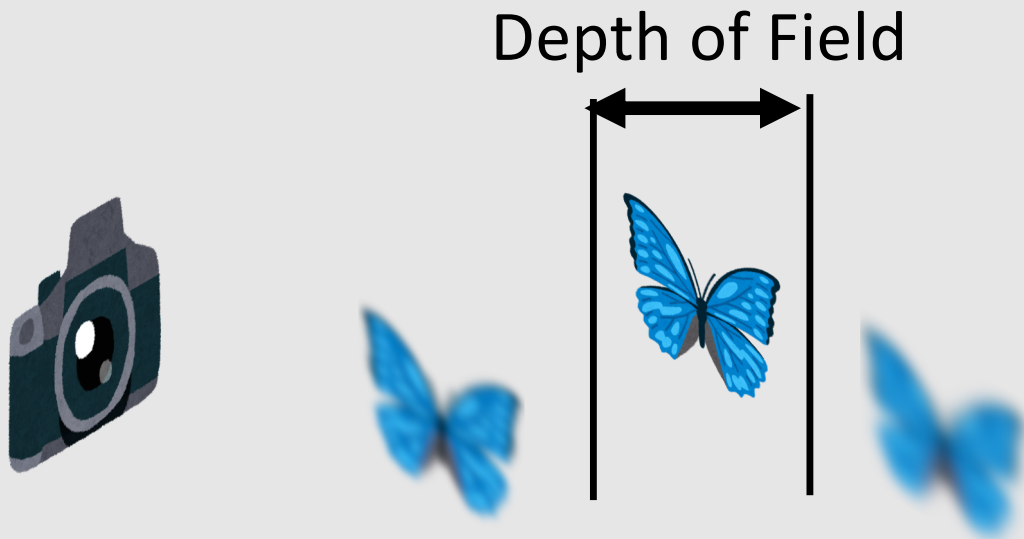
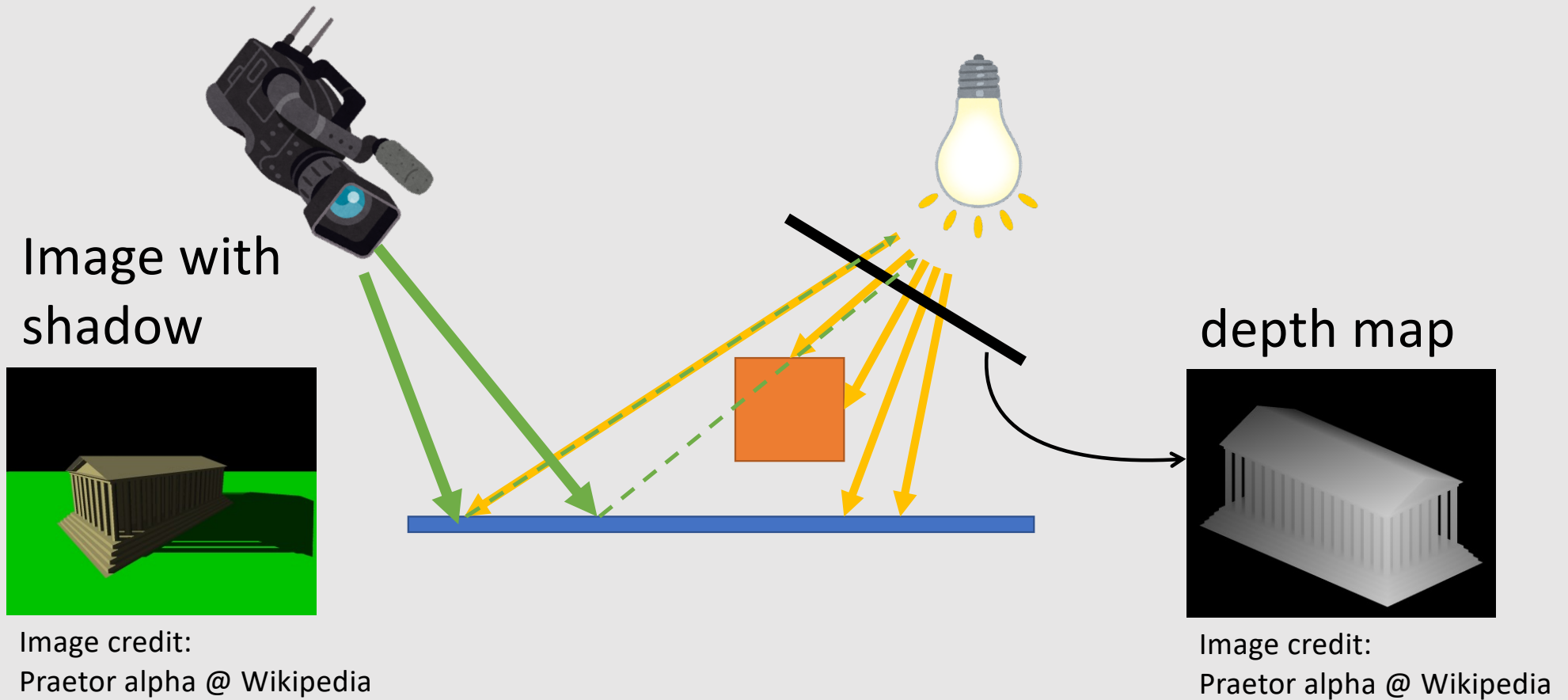


image from wikipedia

- Shallows depth of field → small range of focus, large appature
- Deep depth of field → pan focus, small appature

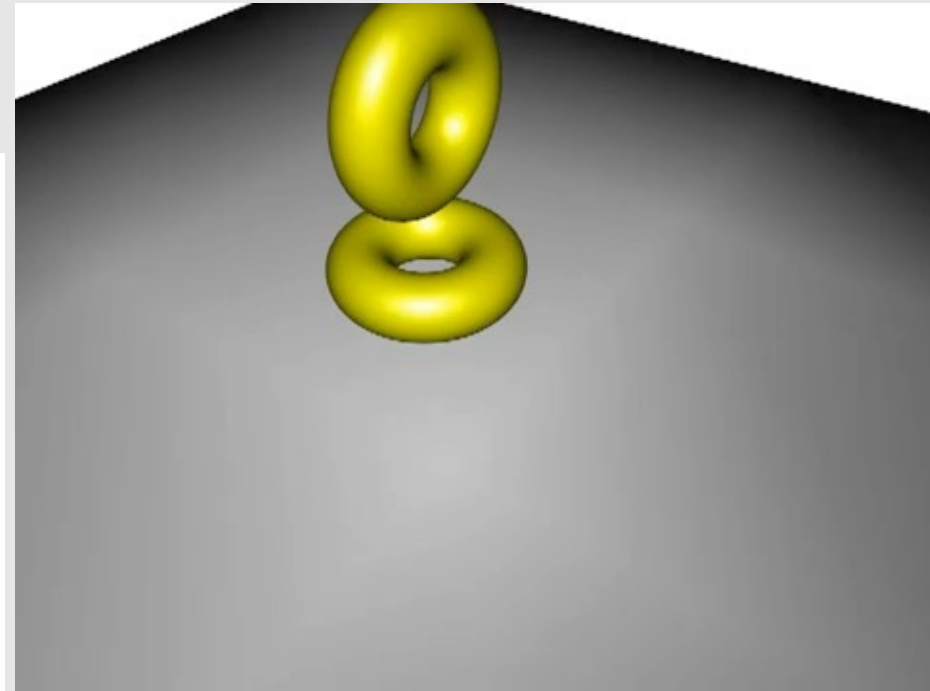
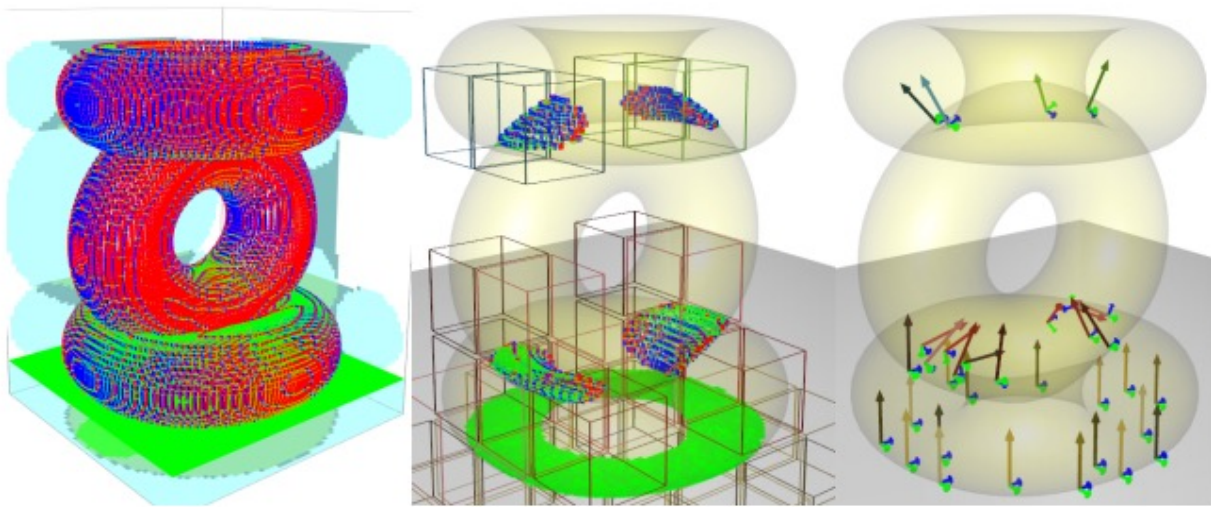
# Depth Image Usage 3: Shadow Mapping

- **Rendering image from light** to find occlusion of light



# Depth Image Usage 4: Collision Detection

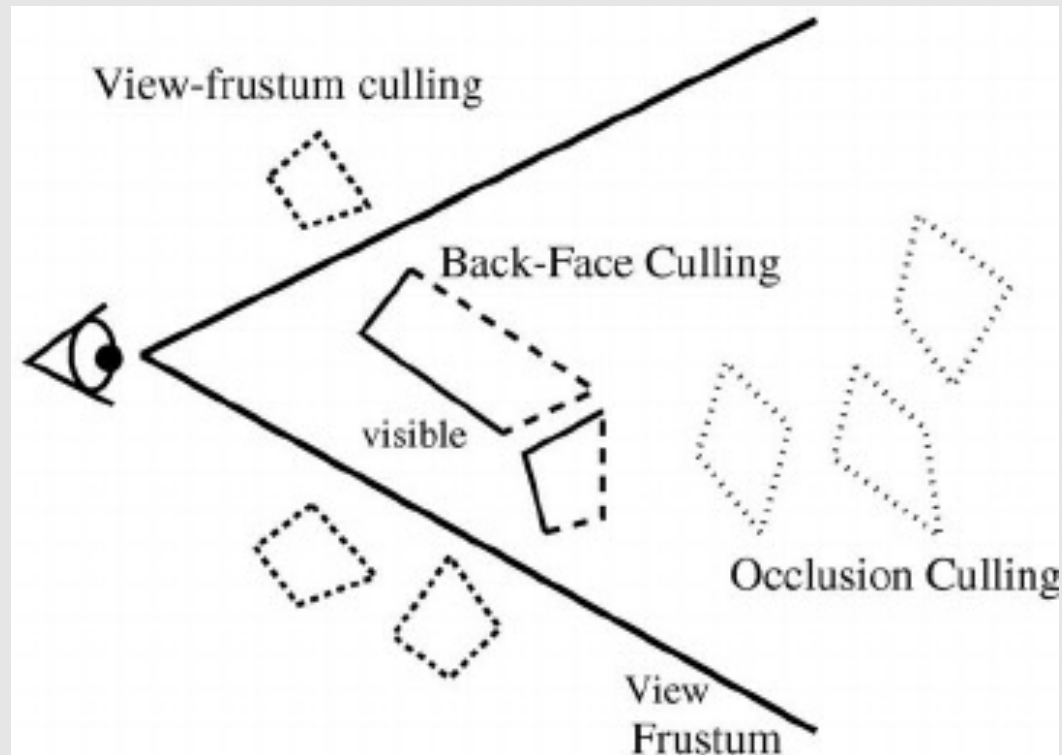
- Compute volume of intersection and its derivative



J r mie Allard, Fran ois Faure, Hadrien Courtecuisse, Florent Falipou, Christian Duriez, and Paul G. Kry. 2010. Volume contact constraints at arbitrary resolution. *ACM Trans. Graph.* 29, 4, Article 82 (July 2010)

# Acceleration Method 1: Culling

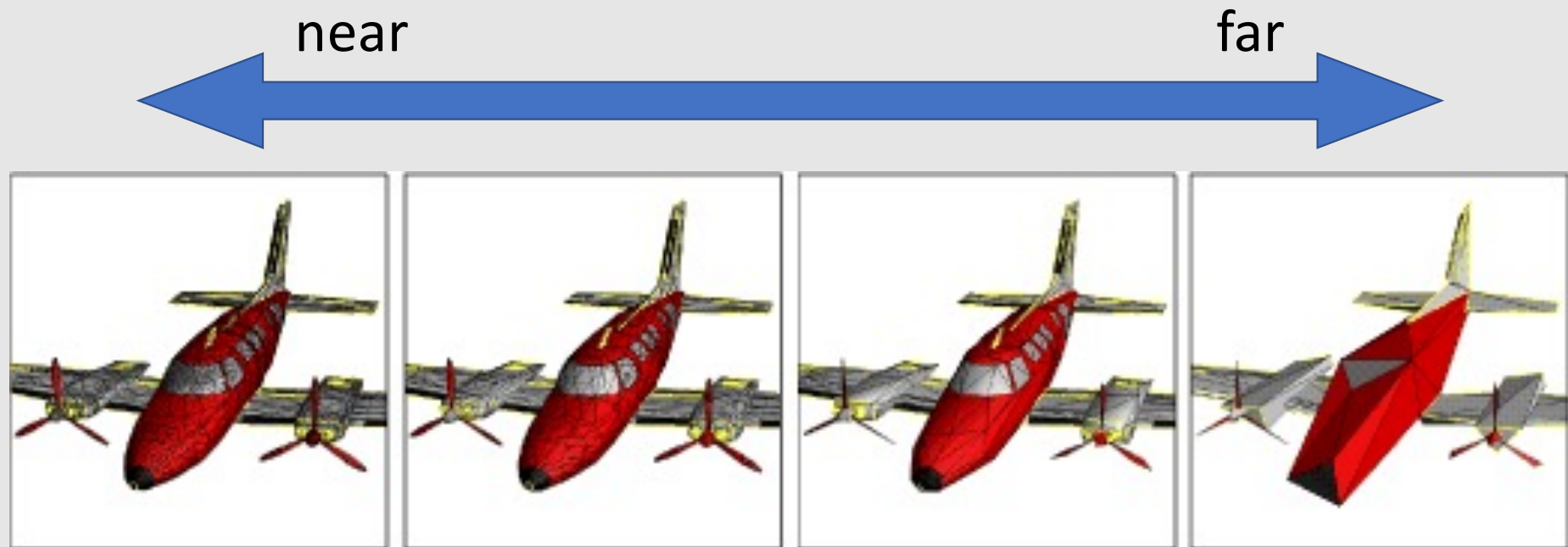
- Reduce number of triangle rasterized



Cohen-Or, Daniel & Chrysanthou, Yiorgos & Silva, Cláudio. (2001). A Survey of Visibility for Walkthrough Applications. Proceedings of SIGGRAPH.

# Acceleration Method 2: Level of Detail (LoD)

- Dynamically change the resolution of mesh



Hoppe, H. Progressive meshes. In Computer Graphics (SIGGRAPH'96 Proceedings).



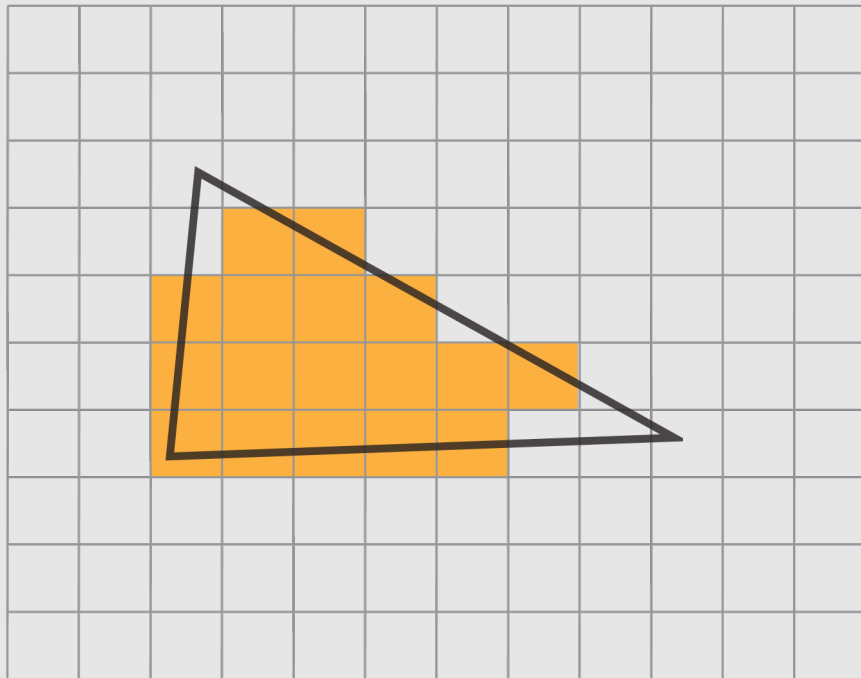
# Nanite in Unreal Engine 5



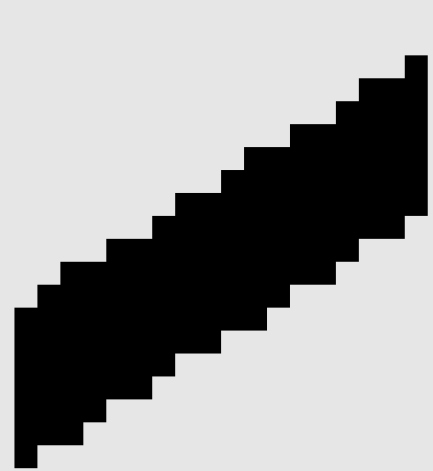
Nanite in UE5: The End of Polycounts? | Unreal Engine  
<https://www.youtube.com/watch?v=xUUSsXswyZM>

# Sub-pixel Effects

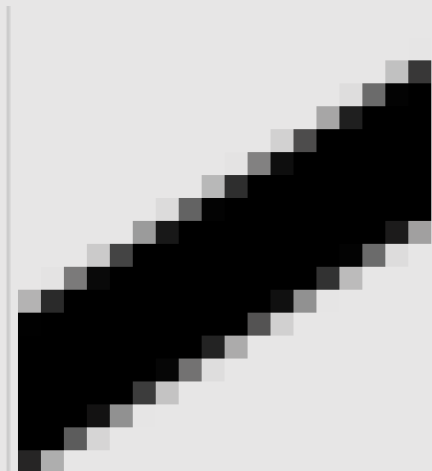
# Removing Jaggy Edge: Anti-Aliasing



aliased

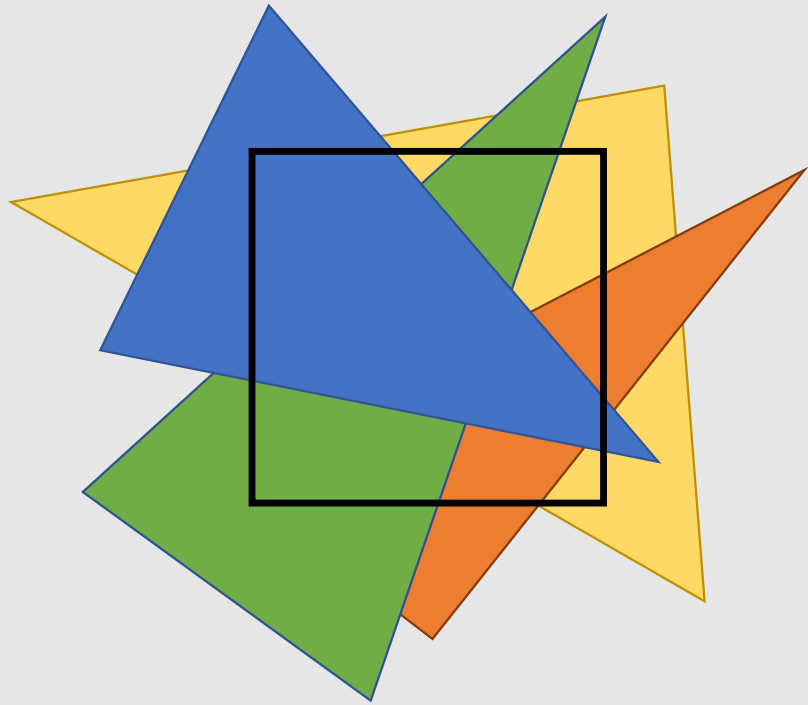




anti-aliased





# How to Compute the “Coverage Ratio”?



What is the area of  visible inside the pixel  ?



# Monte Carlo Integration

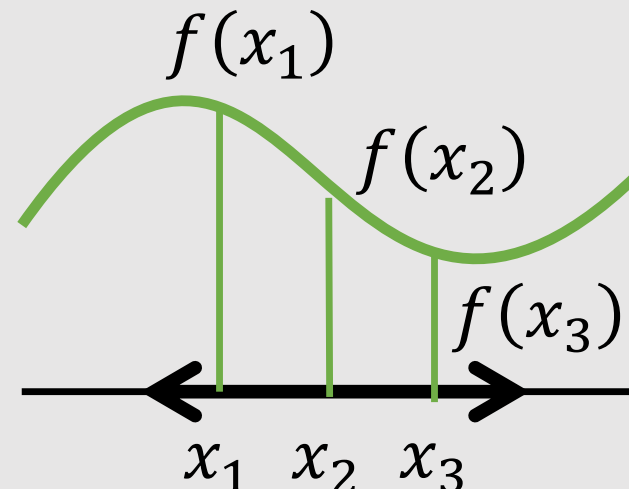
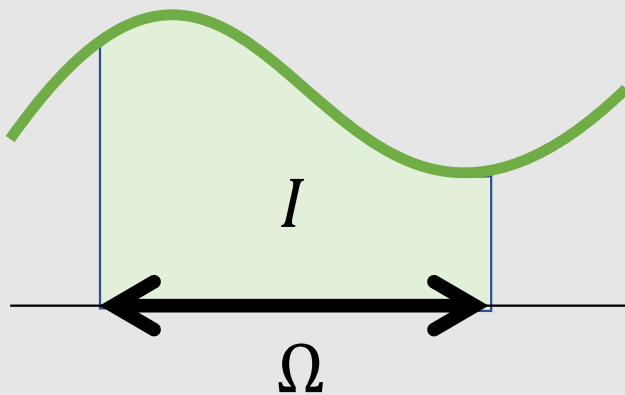
- Integration of a “difficult” function (i.e., we can only evaluate at discrete sample locations)

$$I = \int_{\Omega} f(x) dx$$

approximation

$$\frac{V}{N} \sum_{i=1}^N f(x_i) \quad V = \int_{\Omega} dx$$

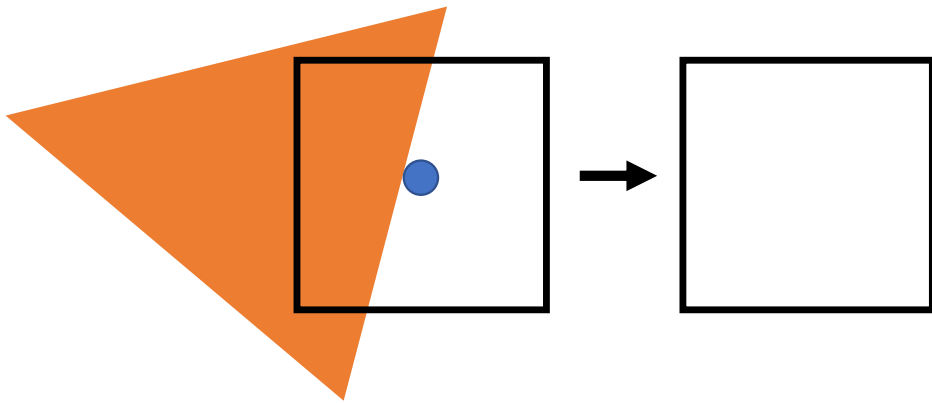
$x_1, \dots, x_N \in \Omega$



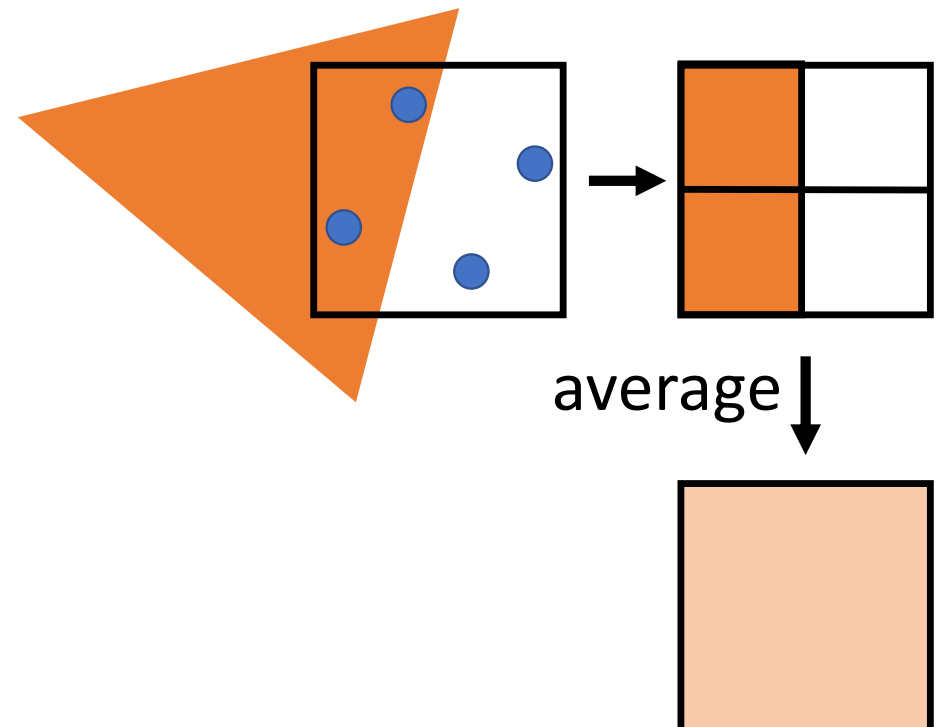
# Basic Approach: Multiple Samples in a Pixel

- Finding coverage ratio approximately

*Sample at center*



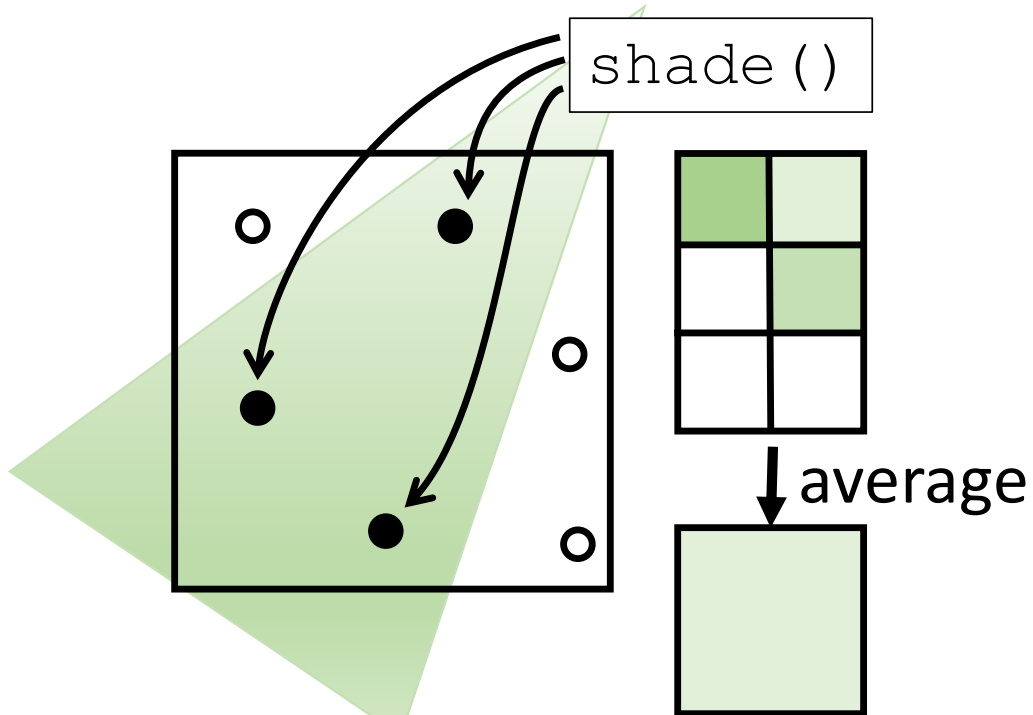
*Multiple samples*



# SuperSampling vs. MultiSampling

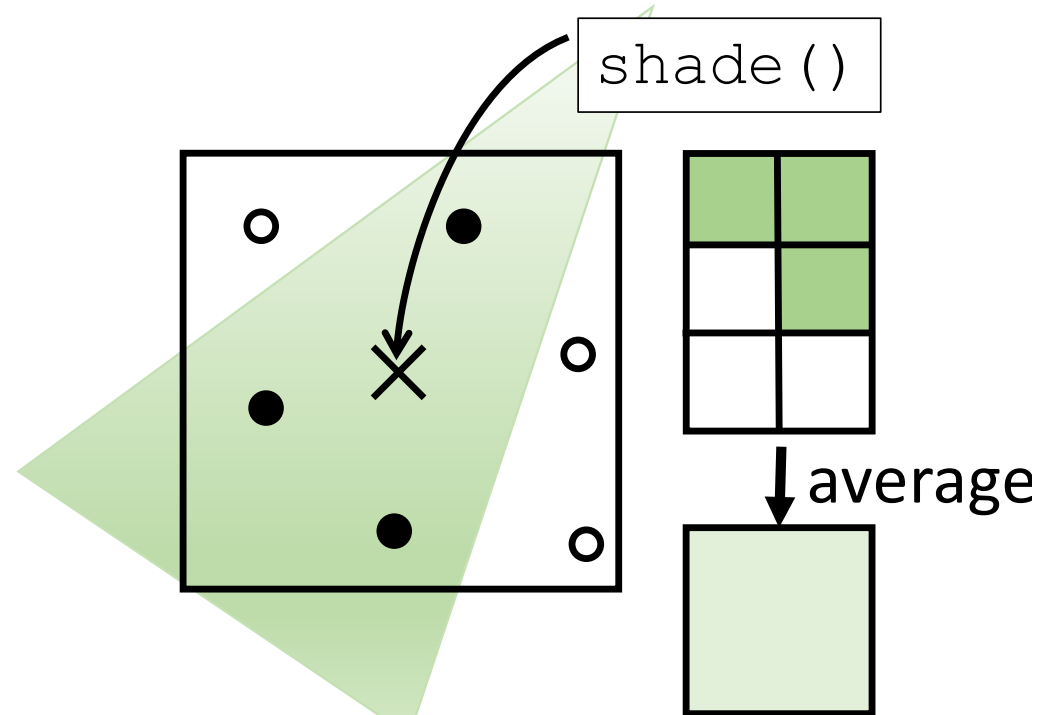
## *SuperSampling*

fragment shader  
for **all** samples



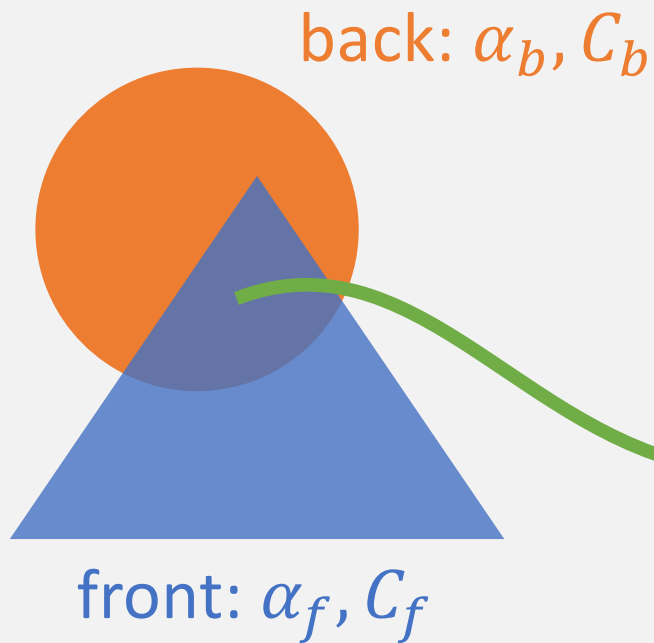
## *MultiSampling*

fragment shader  
for **one** sample

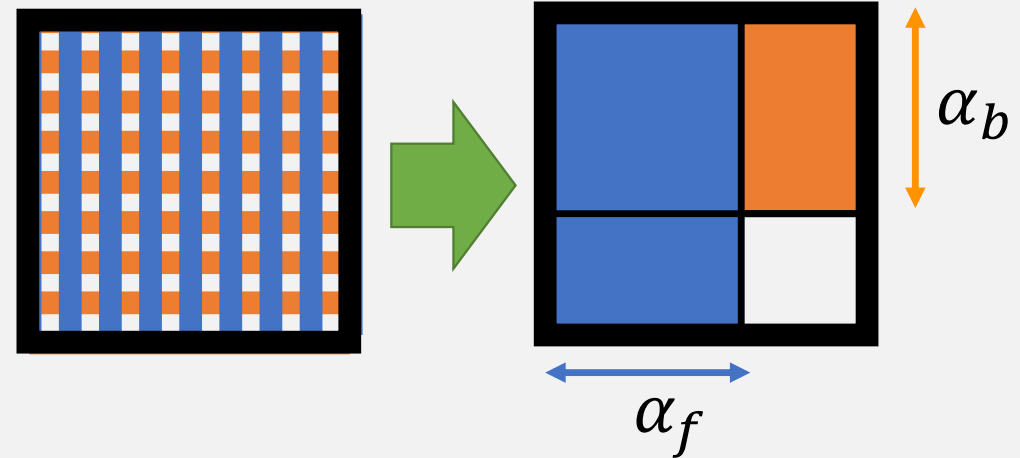


# Transparency is Order Dependent

- Alpha value
  - 0 → completely transparent
  - 1 → opaque



inside one pixel



$$\alpha = \alpha_f + \alpha_b(1 - \alpha_f)$$
$$C\alpha = C_f\alpha_f + C_b\alpha_b(1 - \alpha_f)$$

*Not symmetric!*

# Painter's algorithm

- Sort geometry w.r.t. depth
- Draw from background

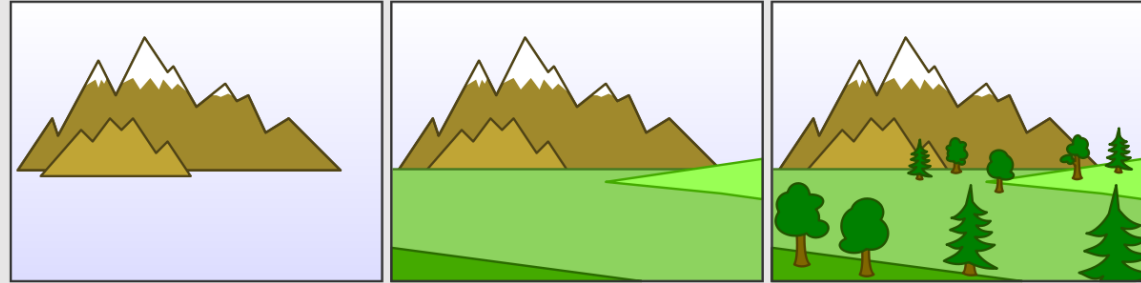
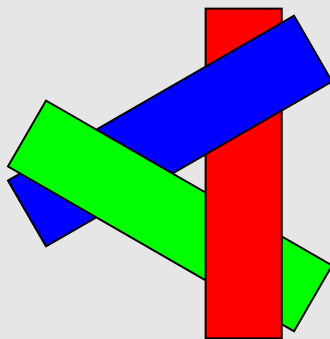


Image Credit: Zapyon @ Wikipedia



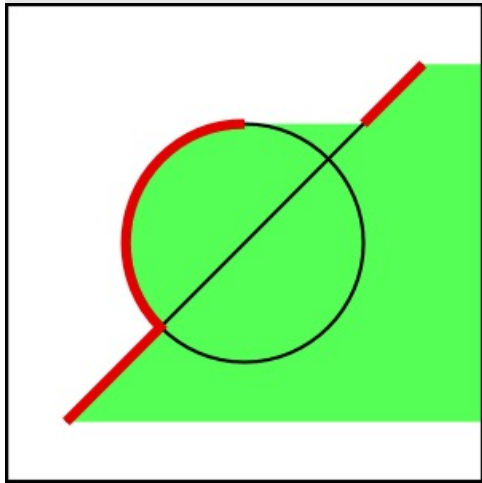
- ☹️ Cannot draw in parallel
- ☹️ Cannot handle cyclical overlapping

Image Credit: Wojciech Muła @ Wikipedia

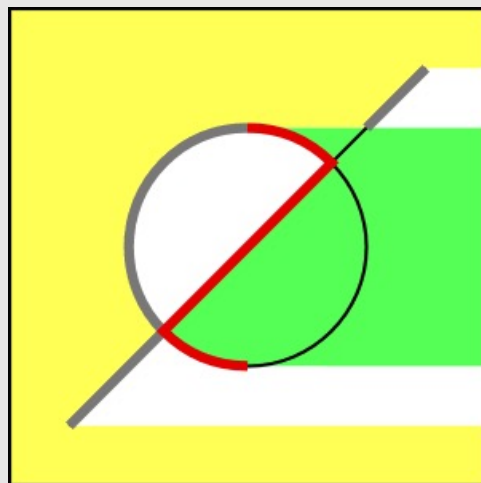
# Depth Peeling Technique [Evenritt et al]

- Use two depth buffer to render object from front to back

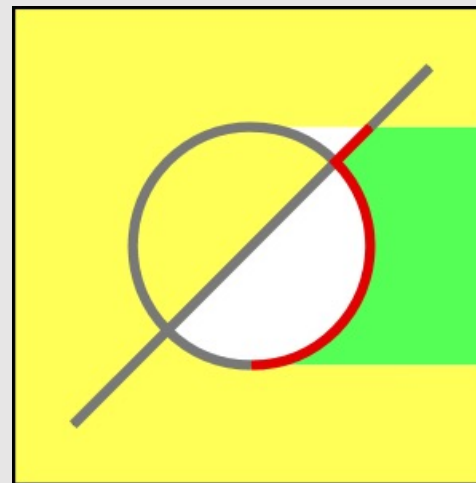
Pass 1



Pass 2



Pass 3



[Evenritt et al]

Image credit: 床井研究室, 2008年11月23日 Depth Peeling  
<https://marina.sys.wakayama-u.ac.jp/~tokoi/?date=20081123>

Everitt, Cass (2001-05-15). "Interactive Order-Independent Transparency" (PDF). Nvidia