Grid-based Fluid Simulation

Stable Fluids [Stam 1999]

- One of the most cited paper in the computrer grahics field

Jos Stam

NVIDIA

Verified email at nvidia.com - Homepage

computer graphics

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Stable fluids J Stam Proceedings of the 26th annual conference on Computer graphics and …	2433	1999
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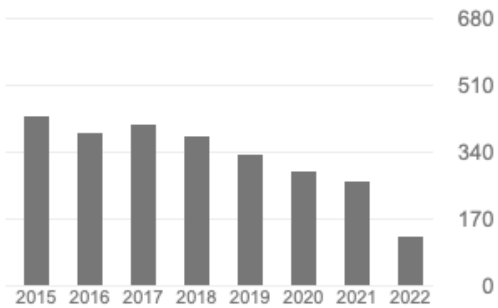

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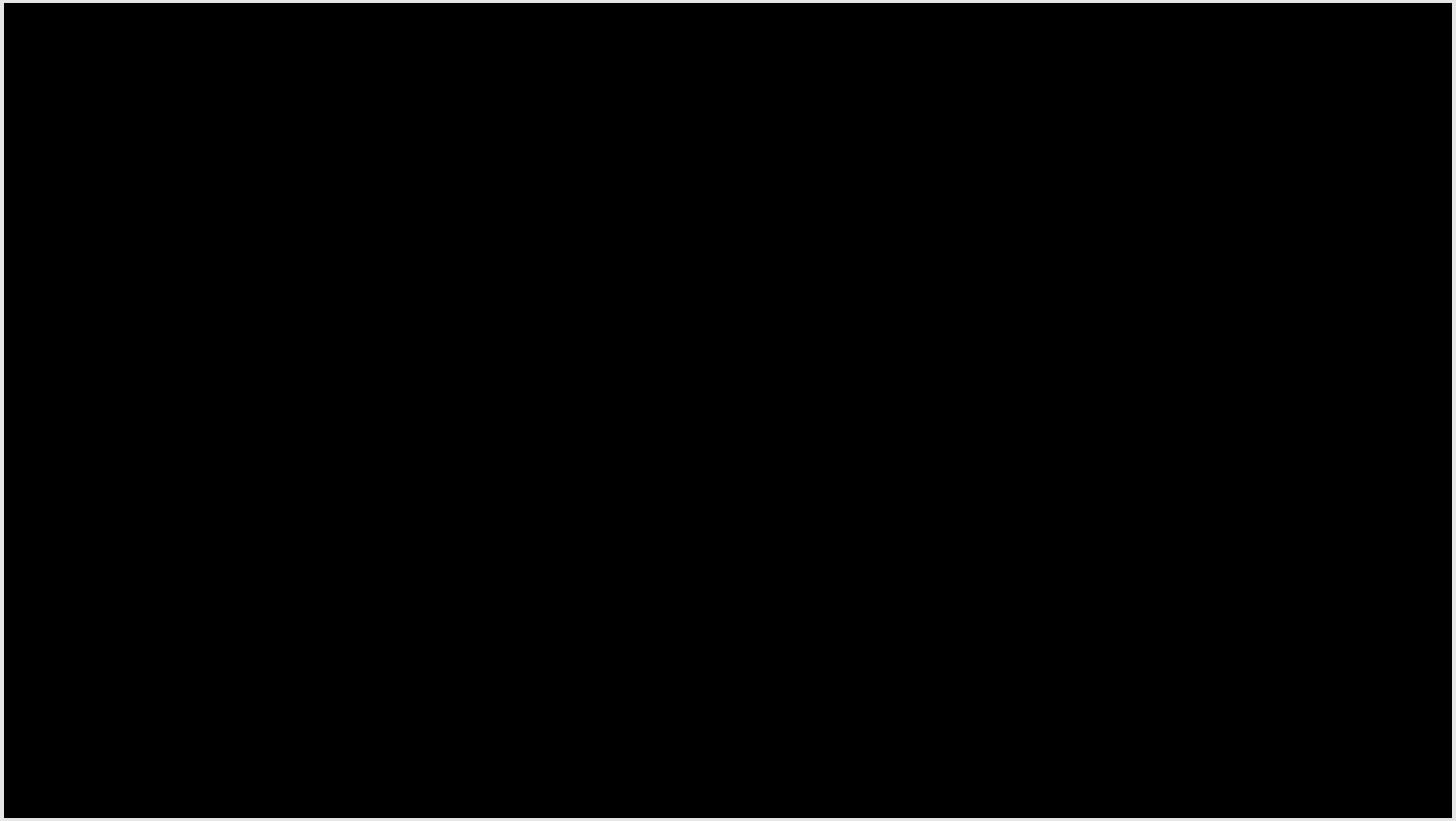
Implementation of Stable Fluids



Magic Fluids - beautiful fluid simulation app for iOS and Android
https://www.youtube.com/watch?v=FdhRi6Zsh5w

Key Ingredients of Fluid

Inertia

Incompressivity

Navier-Stokes equation

$$\rho \left\{ \frac{\partial \phi}{\partial t} + (\vec{v} \cdot \nabla)\phi \right\} = -\nabla p$$

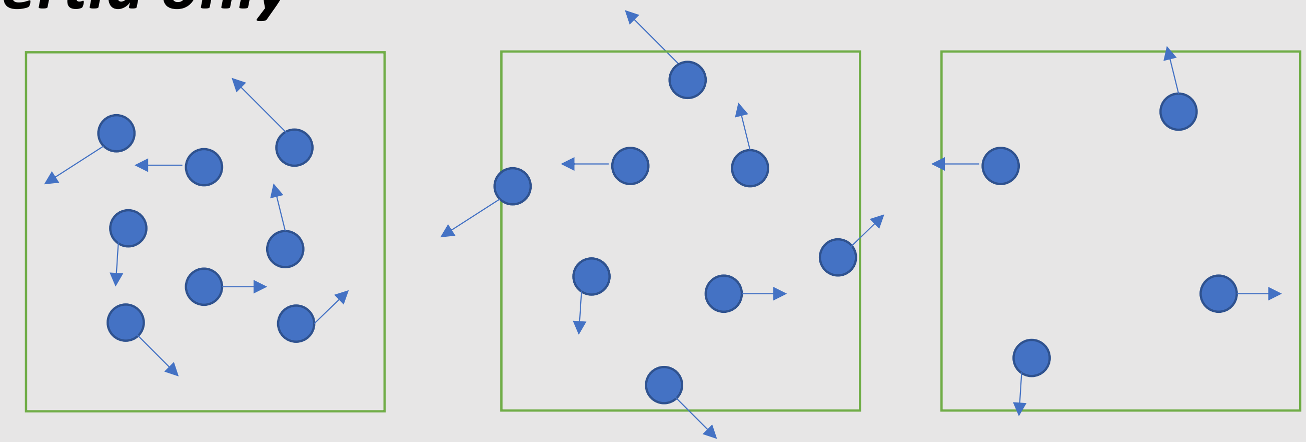
$$\nabla \cdot \vec{v} = 0$$

Incompressibity Makes Vortex

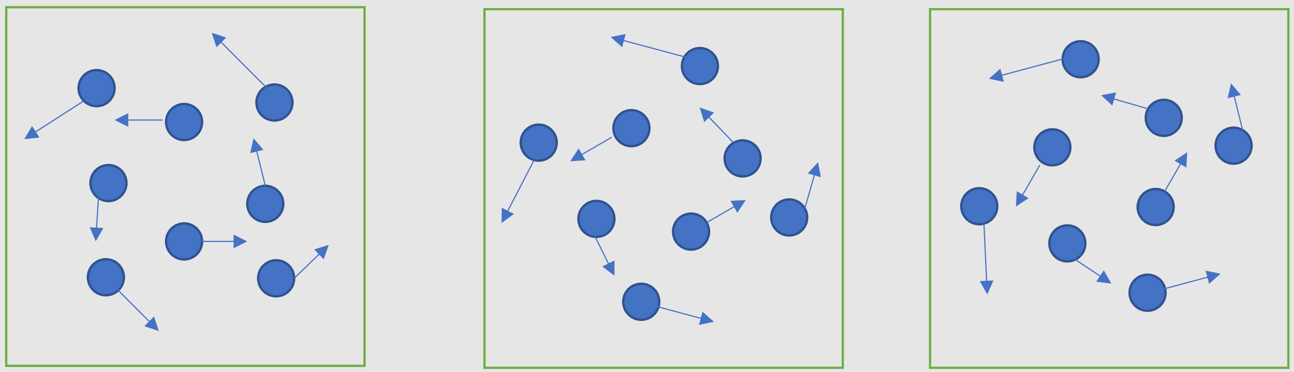
Credit: Astrobob @ Wikipedia

Vortex and Particles

Inertia only

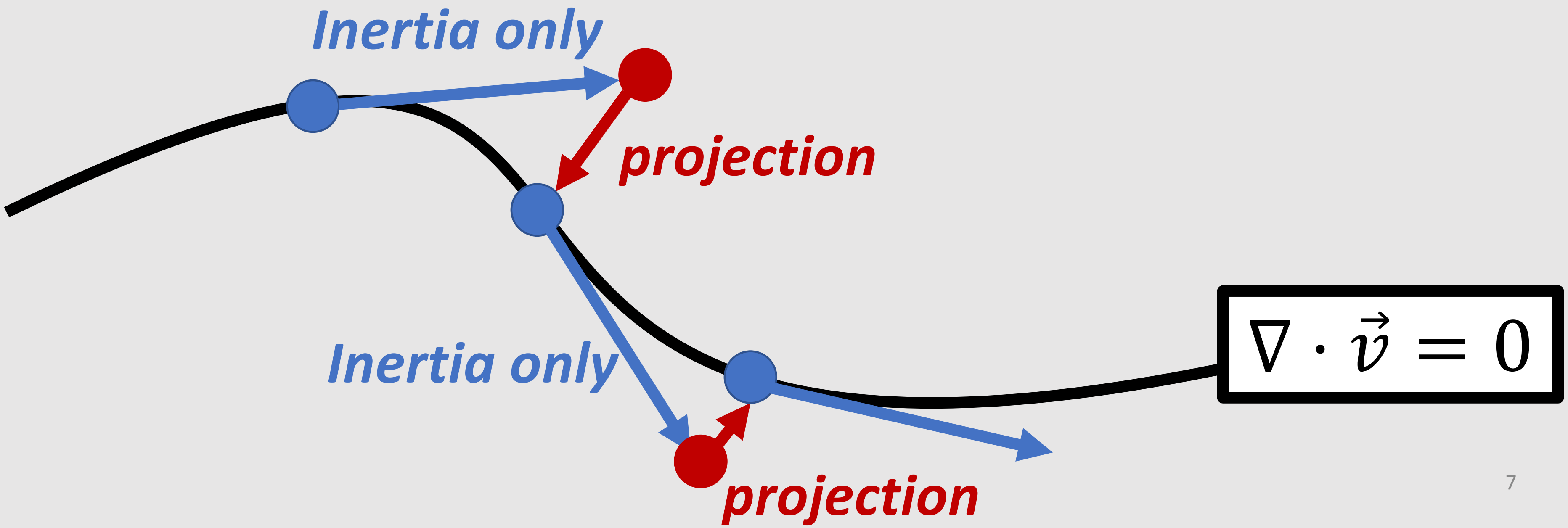


Inertia & incompressivity



Operator Splitting for Fluid Animation

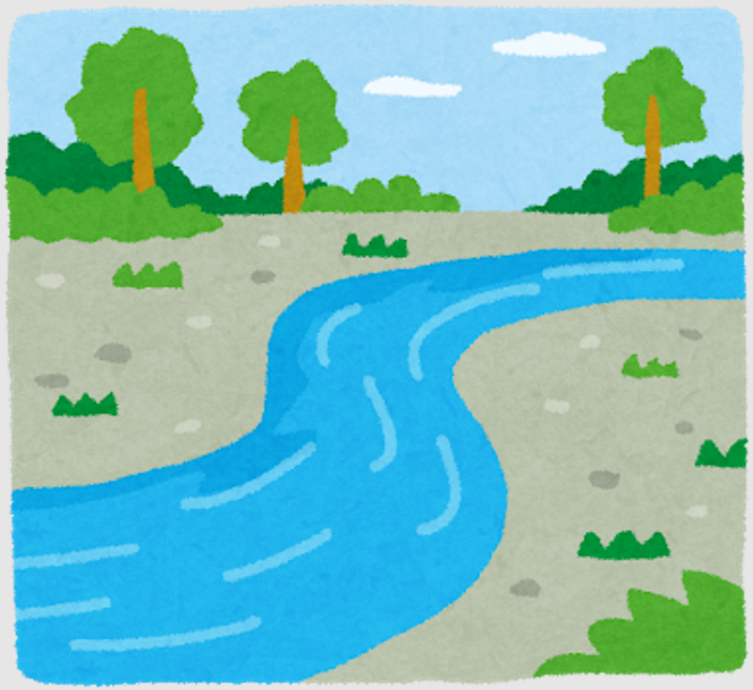
$$\rho \left\{ \frac{\partial \phi}{\partial t} + (\vec{v} \cdot \nabla)\phi \right\} = -\nabla p$$
$$\nabla \cdot \vec{v} = 0$$



Lagrangian vs. Eulerian

Temperature of a River

- How to record the history of temperature of the flowing water?



Reference Frames

Lagrangian

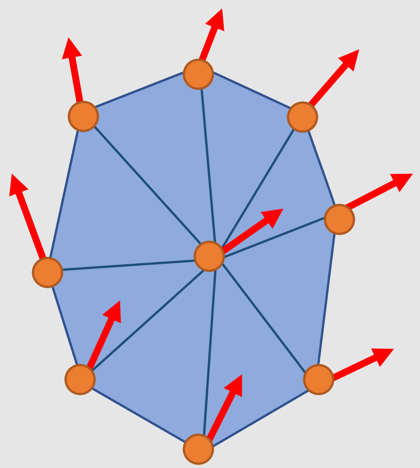
Observation point is moving together with flow

Eulerian

Observation point is fixed

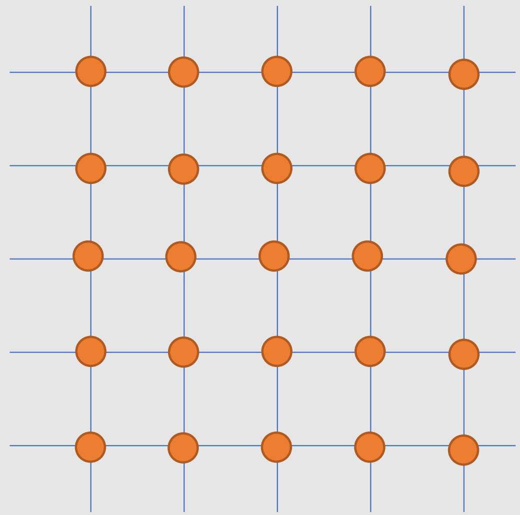
Data Structure for Continuum

Lagrangian
(e.g., deformable mesh)



Observation points moves over time

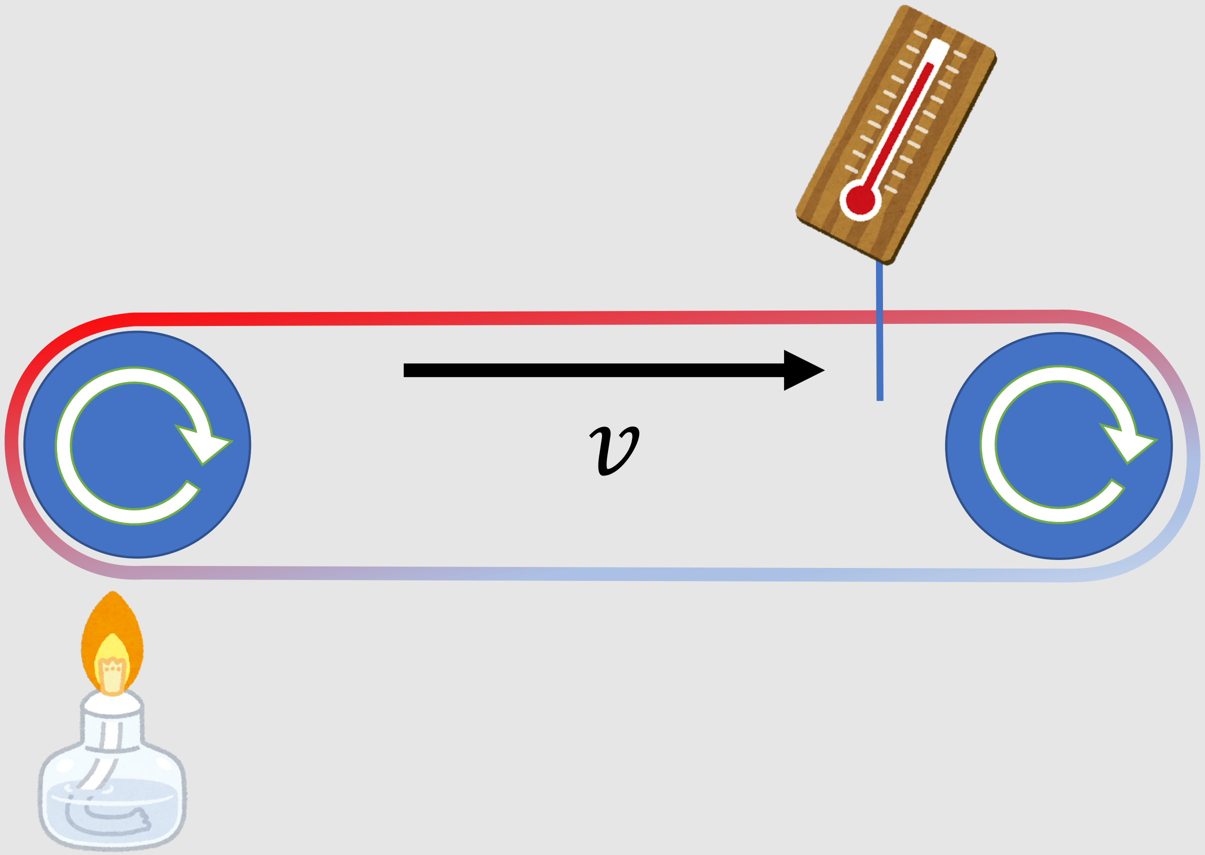
Eulerian
(e.g., regular grid)



Observation points don’t move

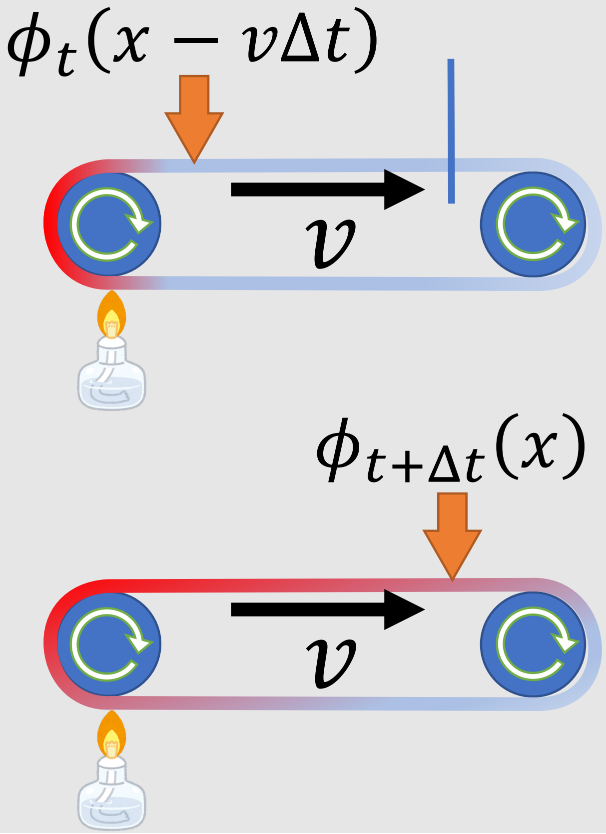
Moving Field Observed at Fixed Position

- Measuring the change of the temperature on the carousel



Material Derivative

- Measuring the change of the temperature on the carousel

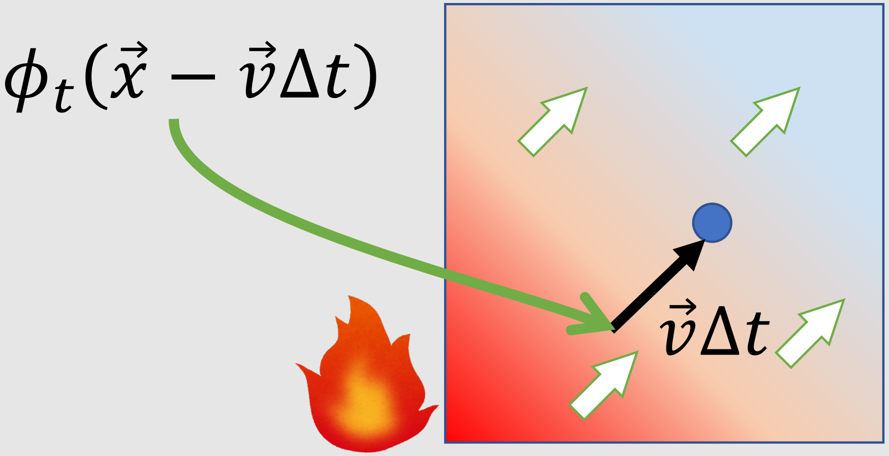


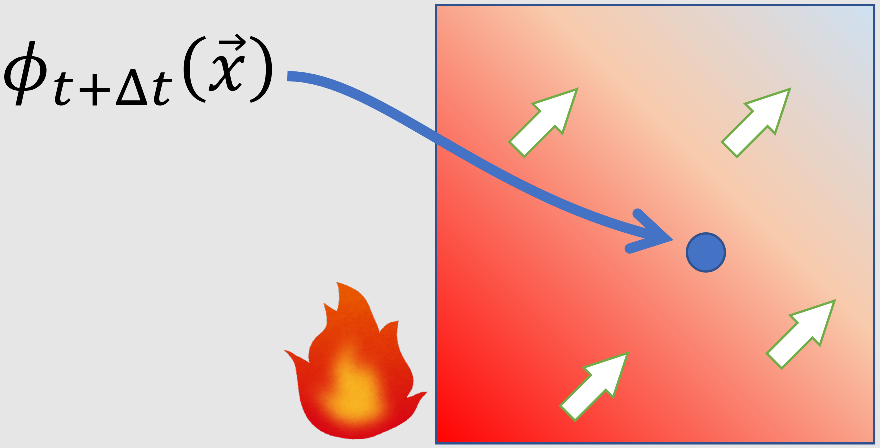
$$\phi_{t+\Delta t}(x) \simeq \underbrace{\phi_t(x - v\Delta t)}_{\simeq \phi_t(x) + \frac{\partial \phi}{\partial x}(-v\Delta t)} + \frac{D\phi}{Dt}\Delta t$$

$\frac{D\phi}{Dt}$: Material derivative

$$\frac{\partial \phi}{\partial t} = -v\frac{\partial \phi}{\partial x} + \frac{D\phi}{Dt}$$

Material Derivative in 2D





$$\phi_{t+\Delta t}(\vec{x}) \simeq \phi_t(\vec{x} - \vec{v}\Delta t) + \frac{D\phi}{Dt}\Delta t$$

$$\simeq \phi_t(x) + \frac{\partial \phi}{\partial x}(-v_x\Delta t) + \frac{\partial \phi}{\partial y}(-v_y\Delta t)$$

$$\frac{\partial \phi}{\partial t} = -(\vec{v} \cdot \nabla)\phi + \frac{D\phi}{Dt}$$

Equation of Motion in Lagrangian Frame

- Newton's second law

$$m\frac{d\vec{v}}{dt} = \vec{F}$$

Equation of Motion in Eulerian Frame

$$m\frac{d\vec{v}}{dt} = \vec{F}$$

$$\rho\frac{D\vec{v}}{Dt} = \vec{\mathrm{f}}$$

Pressure gradient, viscousity, gravity…etc

$$= \frac{\partial\phi}{\partial t} + (\vec{v}\cdot\nabla)\phi$$

Fluid With No External Force

$$\vec{v}_{t+\Delta t}(\vec{x}) \simeq \vec{v}_t(\vec{x} - \vec{v}\Delta t) + \frac{D\vec{v}}{Dt}\Delta t$$

No external force $\rho \frac{D\vec{v}}{Dt} = 0$

$$\vec{v}_{t+\Delta t}(\vec{x}) \simeq \vec{v}_t(\vec{x} - \vec{v}\Delta t)$$

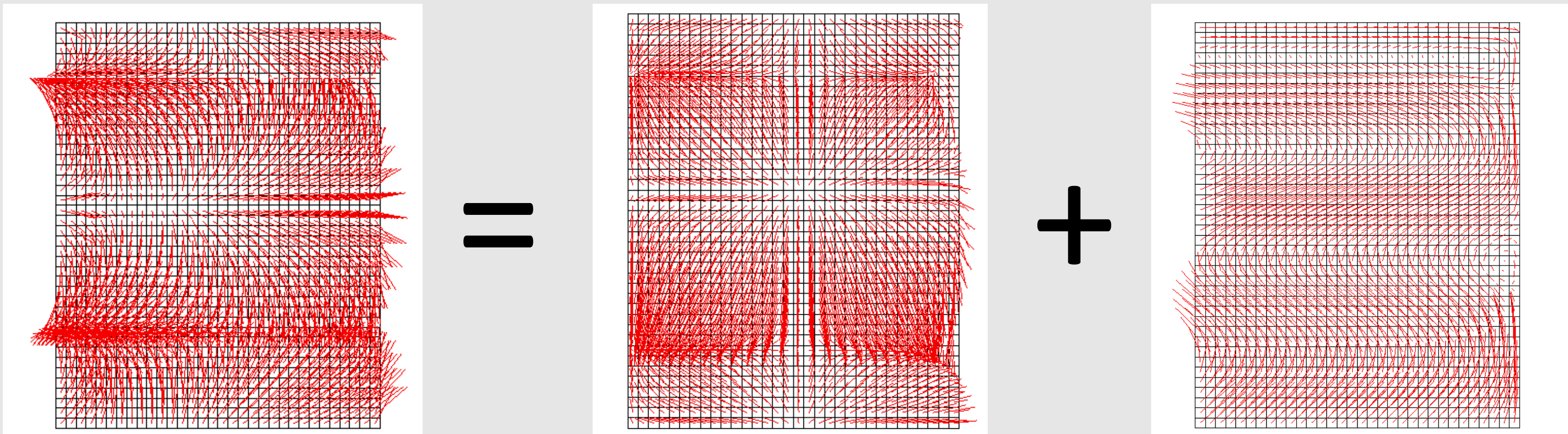
Pressure Projection

Helmholtz Decomposition

$$\vec{v} = \nabla\phi + \nabla\times\Psi$$

No rotation
Only divergence

No divergence
Only rotation



Projection to Incompressive Space

$$\vec{v} = \nabla\phi + \nabla\times\Psi$$

$$\nabla \cdot \vec{v} = \nabla \cdot \nabla\phi + \underbrace{\nabla \cdot (\nabla\times\Psi)}_{= 0}$$

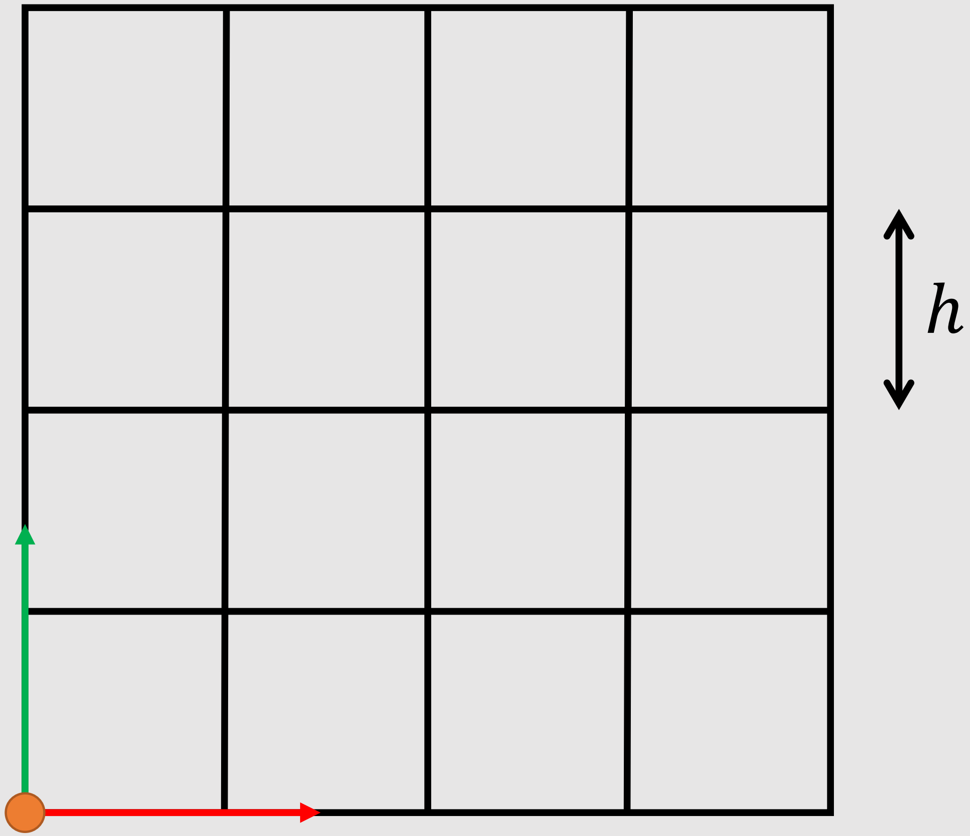
$$\vec{v}' = \vec{v} - \nabla\phi$$

Solution for Poision's equation $\nabla \cdot \nabla\phi = \nabla \cdot \vec{v}$

Spatial Discretization

Regular Grids

- Most common discretization for spatial values

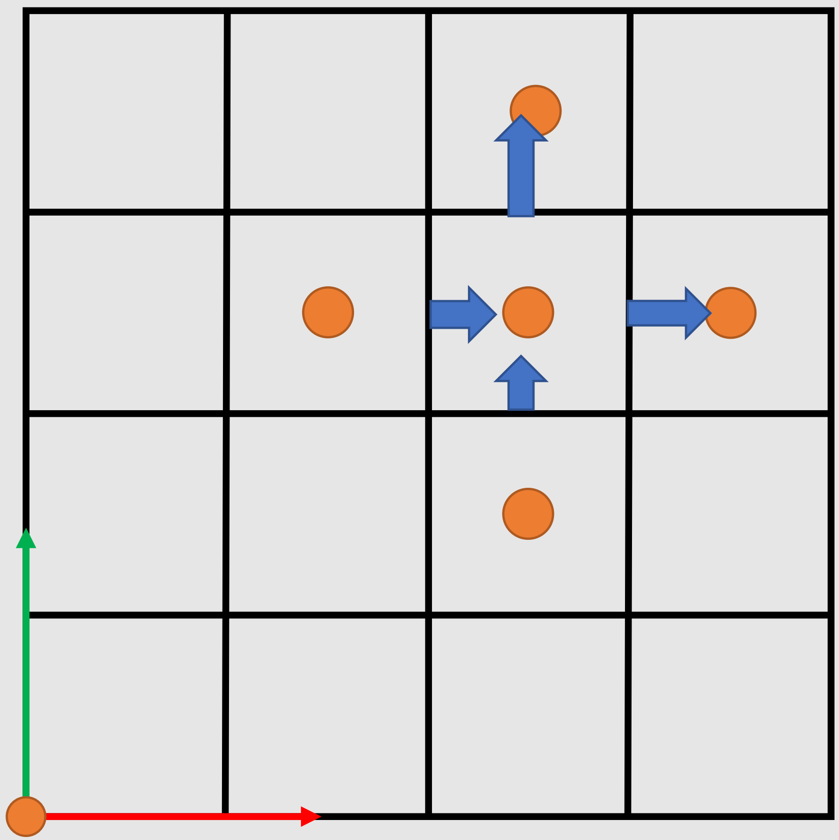


Let’s find out the corresponding grid cell for (p_x, p_y)

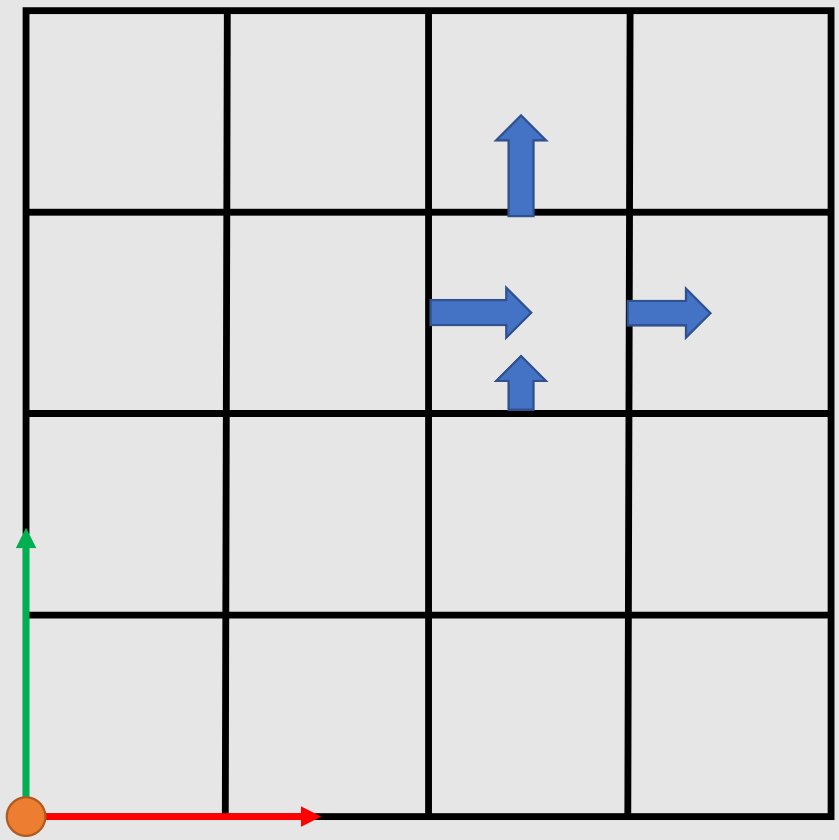
Check it out!

Staggered Grid

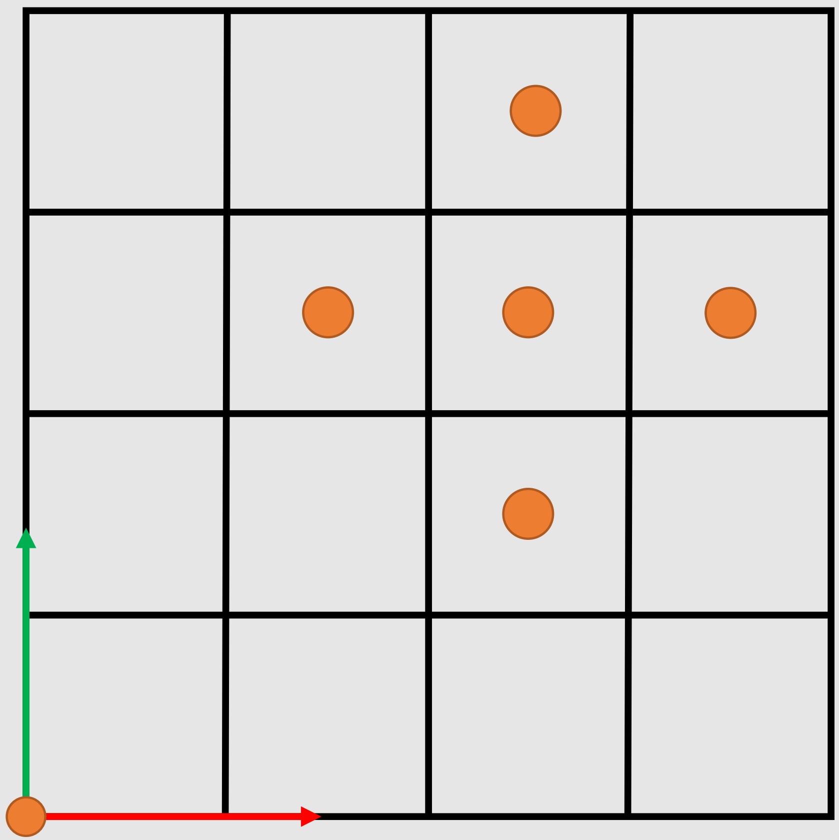
- Most common discretization for grid-based fluid simulation



Staggerd Lattice for Divergence

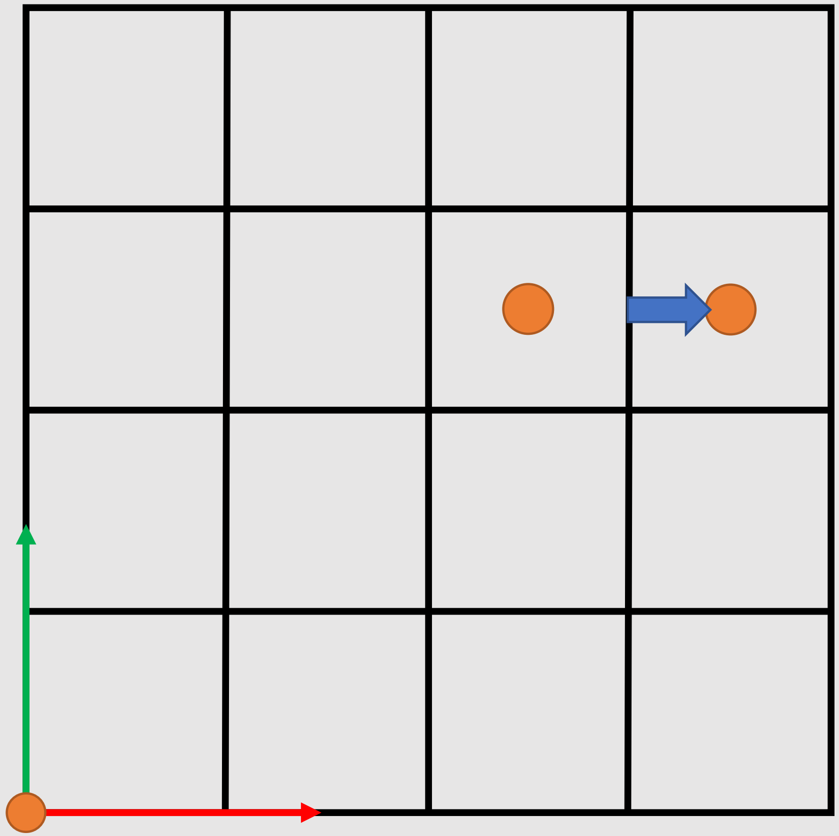


Staggered Grid for Poisson Equation

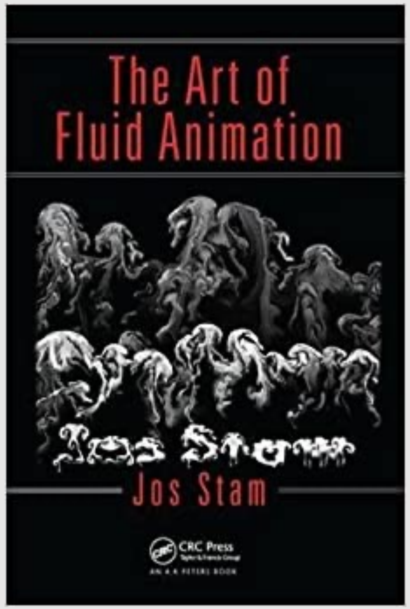


Staggered Grid

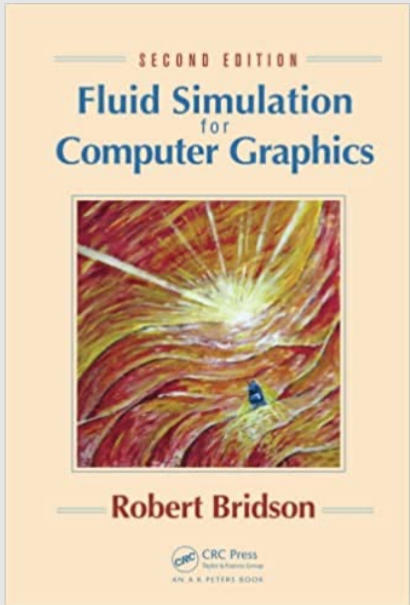
- Most common discretization for grid-based fluid simulation



For Further Study

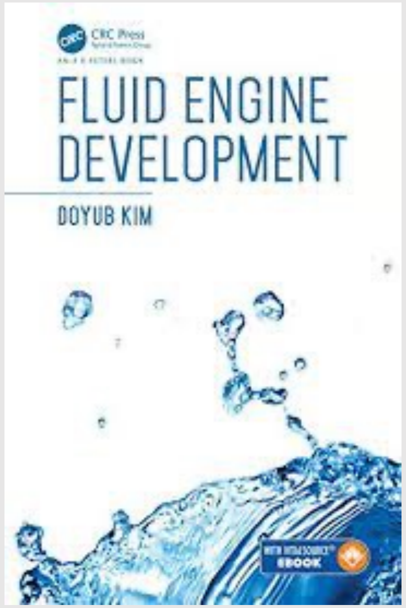


The Art of Fluid Animation



Fluid Simulation for Computer Animation

https://www.cs.ubc.ca/~rbridson/fluidsimulation/



Fluid Engine Development