Barycentric Coordinate （重心座標）

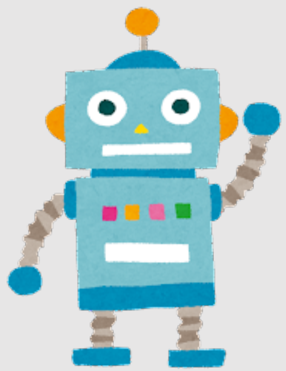
Coordinate on a simplex defined using length/area/volume used for interpolation

What is Coordinate?

- Coordinate is a parameterization of position

parameters

$$(w_1, w_2 \dots)$$

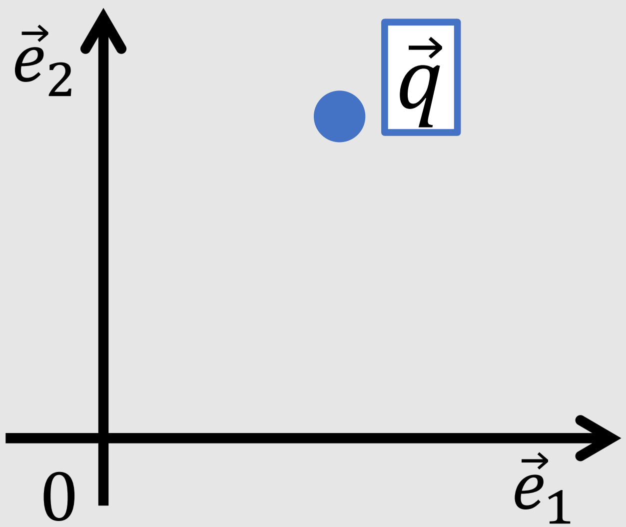


position

$$\vec{q}$$

Cartesian Coordinate (デカルト座標系)

- Origin and orthonomal basis vectors

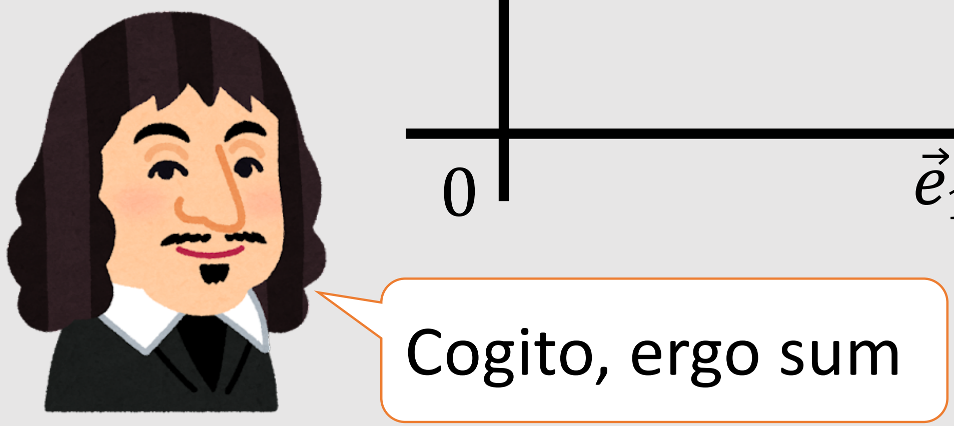


Given coordinate (w_1, w_2) :

$$\vec{q} = w_1\,\vec{e}_1 + w_2\vec{e}_2$$

Given position $\vec{q}$

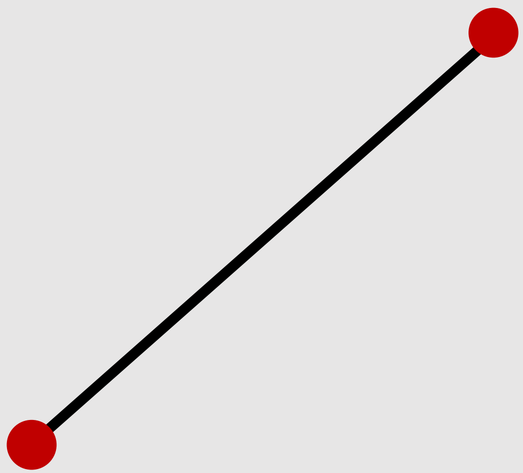
$$w_1 = \vec{q}\cdot\vec{e}_1, \qquad w_2 = \vec{q}\cdot\vec{e}_2$$



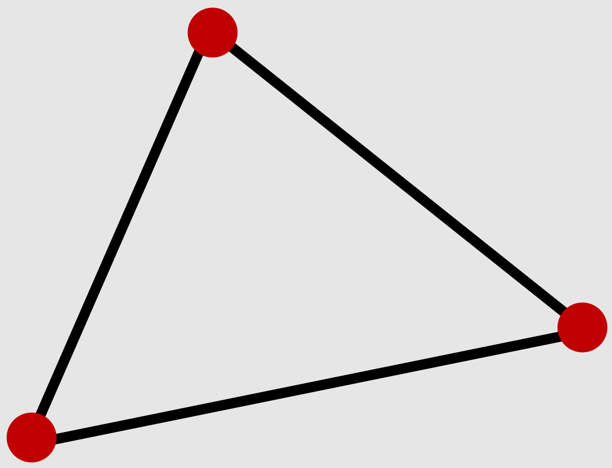
René Descartes

What is Simplex?

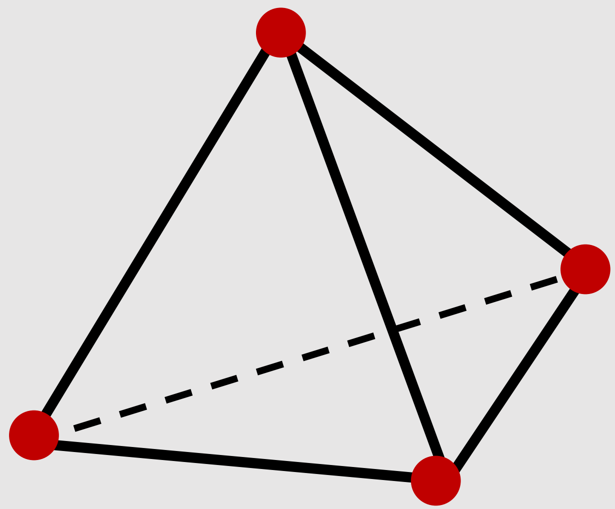
- Simplest polytope (i.e. shape with flat sides)



1-simplex:
line segment

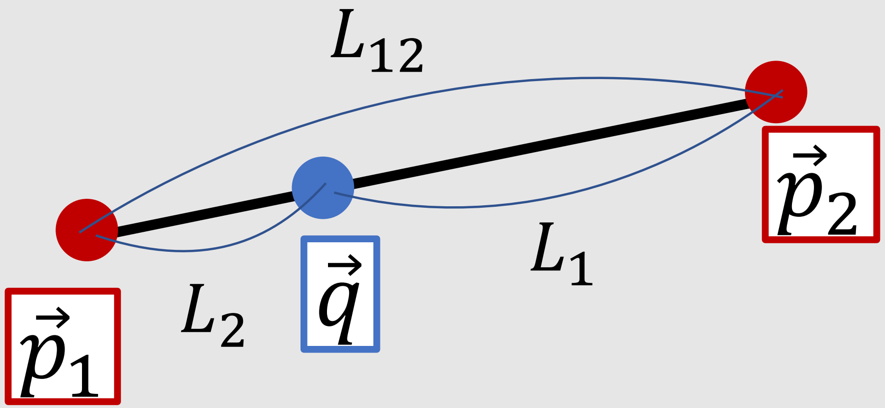


2-simplex:
triangle



3-simplex:
tetrahedron

Barycentric Coordinate on Line



Given coordinate (w_1, w_2)

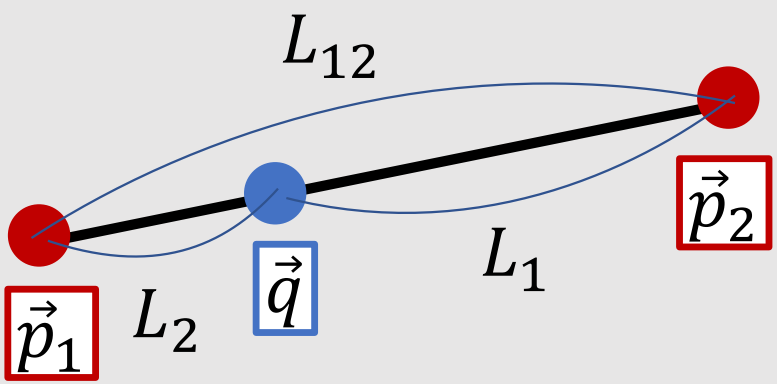
$$\vec{q} = w_1 \, \vec{p}_1 + w_2 \vec{p}_2$$

Given position $\vec{q}$

$$w_1 = \frac{L_1}{L_{12}}, w_2 = \frac{L_2}{L_{12}}$$

$$w_1 + w_2 = 1$$

Linear Length Computation on Line



Naïve length :

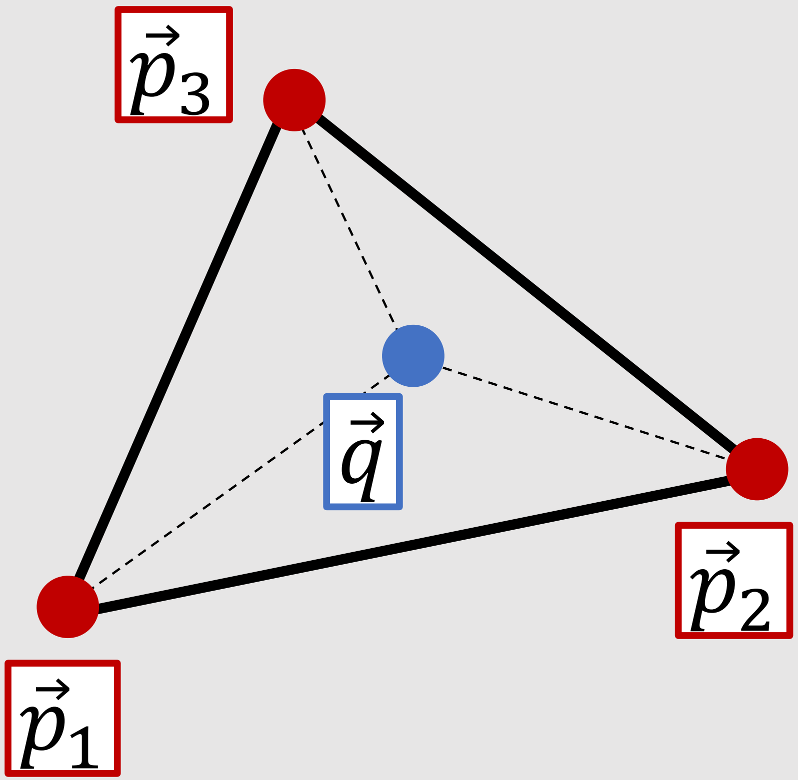
$$L_1 = \|\vec{p}_2 - \vec{q}\|$$

Cannot exterpolate outside line

Linear length :

$$L_1 = (\vec{p}_2 - \vec{q}) \cdot \frac{\vec{p}_2 - \vec{p}_1}{\|\vec{p}_2 - \vec{p}_1\|}$$

Barycentric Coordinate on Triangle



Given coordinate (w_1, w_2, w_3) :

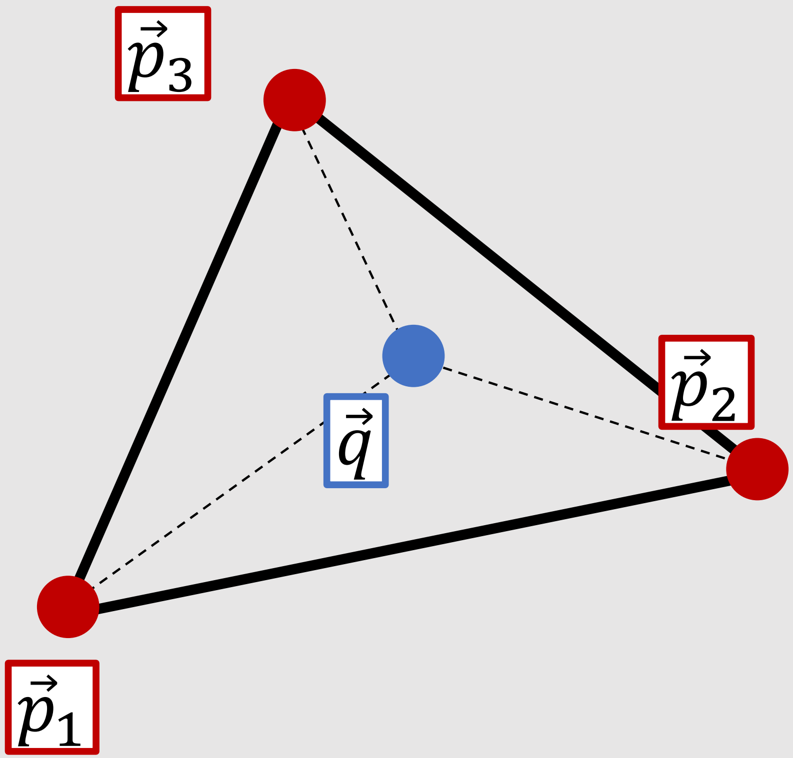
$$\vec{q} = w_1 \vec{p}_1 + w_2 \vec{p}_2 + w_3 \vec{p}_3$$

Given position $\vec{q}$:

$$w_1 = \frac{A_{q23}}{A_{123}}, w_2 = \frac{A_{1q3}}{A_{123}}, w_3 = \frac{A_{12q}}{A_{123}}$$

$$w_1 + w_2 + w_3 = 1$$

Linear Area Computation on Triangle



Naïve area :

$$A_{q23} = \|(\vec{p}_3 - \vec{p}_2)\times(\vec{q} - \vec{p}_2)\|$$

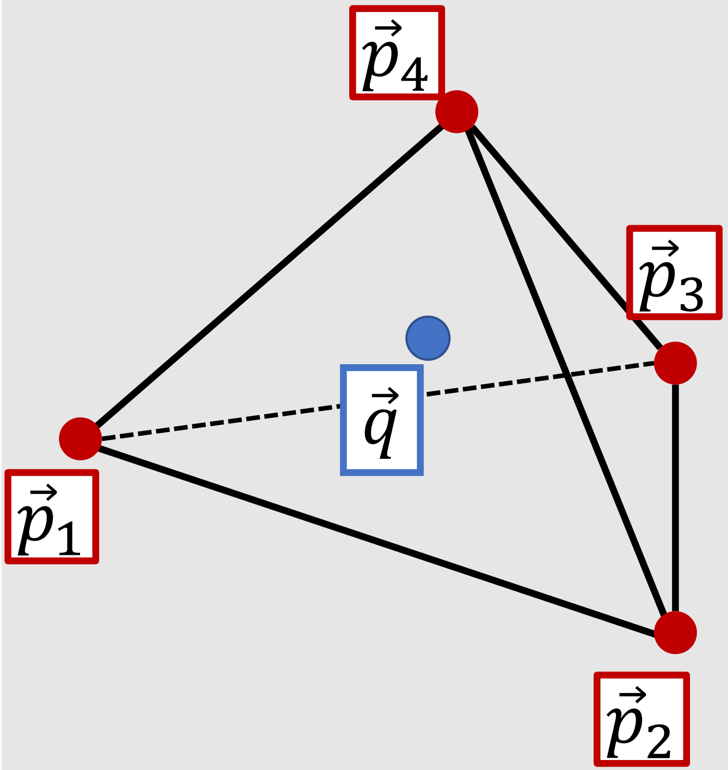
Cannot exterpolate outside triangle

Linear area :

$$A_{q23} = [(\vec{p}_3 - \vec{p}_2)\times(\vec{q} - \vec{p}_2)] \cdot \vec{n}$$

Unit normal vector

Barycentric Coordinates for a Tetrahedron



Given coordinate (w_1, w_2, w_3) :

$$\vec{q} = w_1 \, \vec{p}_1 + w_2 \vec{p}_2 + w_3 \vec{p}_3 + w_4 \vec{p}_4$$

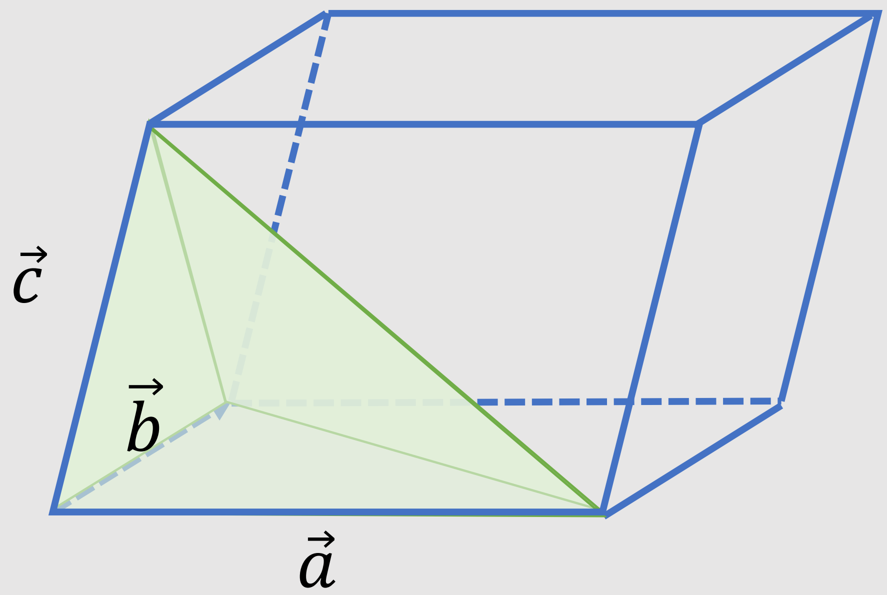
Given position $\vec{q}$:

$$w_1 = \frac{V_{q234}}{V_{1234}}, w_2 = \frac{V_{1q34}}{V_{1234}}, w_3 = \frac{V_{12q4}}{V_{1234}}, w_3 = \frac{V_{123q}}{V_{1234}}$$

$$w_1 + w_2 + w_3 + w_4 = 1$$

Volume of Tetrahedron from Parallelepiped

- Volume of parallelepiped: $\mathrm{V} = \vec{a} \cdot (\vec{b} \times \vec{c})$
- Volume of tetrahedron: $\mathrm{V} = 1/6\, \vec{a} \cdot (\vec{b} \times \vec{c})$

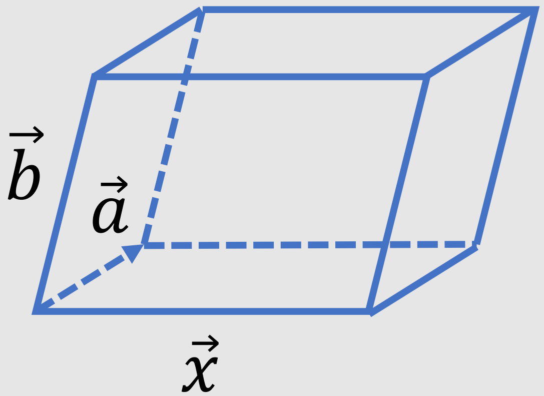


Differential of Scalar Triple Product

$$W(\vec{x}) = \vec{b} \cdot (\vec{a} \times \vec{x})$$

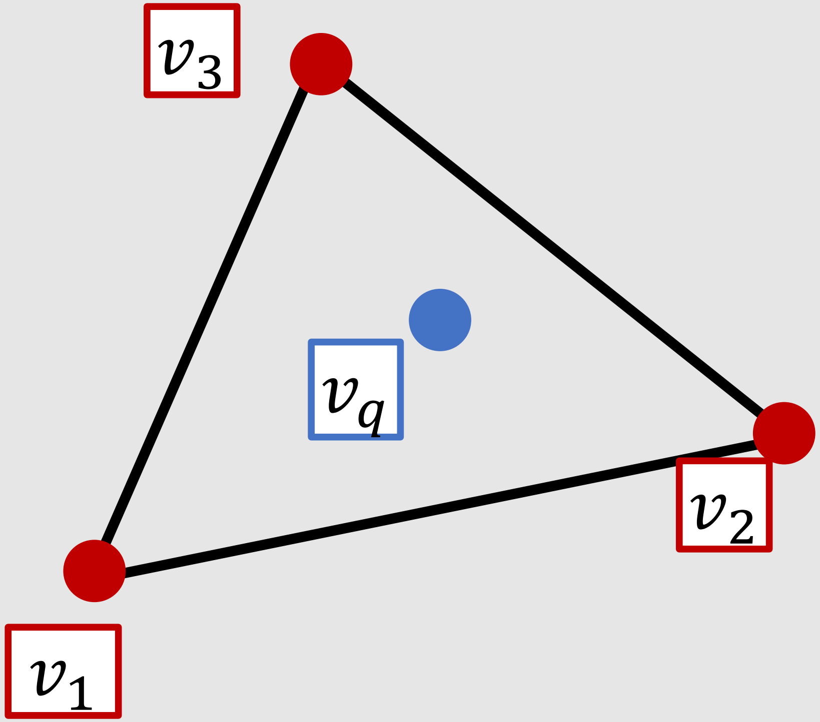
$$= \vec{x} \cdot \left(\vec{b} \times \vec{a}\right)$$

$$\frac{\partial W}{\partial \vec{x}} = \vec{b} \times \vec{a}$$



The volume is linear to the position

Interpolation Using Barycentric Coordinate



Position from coordinate :

$$\vec{q} = w_1 \vec{p}_1 + w_2 \vec{p}_2 + w_3 \vec{p}_3$$

Value from coordinate:

$$v_q = w_1 v_1 + w_2 v_2 + w_3 v_3$$

Interpolation Using Barycentric Coordinate

- Interpolated value is linear w.r.t coordinate